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Simplifying superstring and D-brane actions in $AdS_4 \times CP^3$ superbackground

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ABSTRACT: By making an appropriate choice for gauge fixing kappa-symmetry we obtain a relatively simple form of the actions for a $D = 11$ superparticle in $AdS_4 \times S^7/Z_k$, and for a D0-brane, fundamental string and D2-branes in the $AdS_4 \times CP^3$ superbackground. They can be used to study various problems of string theory and the AdS_4/CFT_3 correspondence, especially in regions of the theory which are not reachable by the $OSp(6|4)/U(3) \times SO(1,3)$ supercoset sigma-model. In particular, we present a simple form of the gauge-fixed superstring action in $AdS_4 \times CP^3$ and briefly discuss issues of its T-dualization.

KEYWORDS: Superstrings and Heterotic Strings, D-branes, AdS-CFT Correspondence, Supergravity Models

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1 Introduction

Recent developments, initiated in [1, 2], which led to important progress in understanding the holographic duality between $D = 3$ superconformal theories and type IIA string/M-theory on AdS_4 have revived an interest in studying strings and branes in supergravity backgrounds whose bosonic subspace is $AdS_4 \times M^6$ and $AdS_4 \times M^7$, respectively, where M^6 is a compactified manifold of $D = 10$ type IIA supergravity and M^7 is its Hopf fibration counterpart in $D = 11$ supergravity (or M-theory). Examples of interest include the supergravity solutions with $M^6 = CP^3$ and $M^7 = S^7/Z_k$ (with an integer k being the Chern-Simons theory level) and their squashings.

In particular, the $\mathcal{N} = 6$ Chern-Simons theory with the gauge group $U(N)_k \times U(N)_{-k}$ [3] has been conjectured to describe, from the CFT_3 side, M-theory on $AdS_4 \times S^7/Z_k$. In the limit of the parameter space of the ABJM theory in which the 't Hooft coupling $\lambda = N/k$ is $\lambda^{5/2} \ll N^2$ and $k \gg 1$, the bulk description is given in terms of perturbative type IIA string theory on the $AdS_4 \times CP^3$ background. To analyze this new type of holographic correspondence from the bulk theory side, an explicit form of the action for the superstring in $AdS_4 \times CP^3$ superspace is required.

In contrast to e.g. the case of type IIB string theory in $AdS_5 \times S^5$ superspace which preserves the maximum number of 32 supersymmetries and is thus described by the supercoset $SU(2, 2|4)/SO(1, 4) \times SO(5)$, the case of type IIA string theory on $AdS_4 \times CP^3$ is more complicated since $AdS_4 \times CP^3$ preserves only 24 of 32 supersymmetries. As a consequence, the complete type IIA superspace with 32 fermionic coordinates, that solves the IIA supergravity constraints for the $AdS_4 \times CP^3$ vacuum solution, is not a coset superspace. This superspace has been constructed in [4] by dimensional reduction of the $AdS_4 \times S^7/Z_k$ solution of $D = 11$ supergravity described by the supercoset $OSp(8|4)/SO(7) \times SO(1, 3) \times Z_k$ with 32 fermionic coordinates. The construction of [4] has generalized to superspace the results of [5–7] on the relation of $AdS_4 \times M^6$ solutions of $D = 10$ type IIA supergravity and $AdS_4 \times M^7$ solutions of $D = 11$ supergravity by identifying the compact manifolds M^7 as S^1 Hopf fibrations over corresponding M^6 .

In [4] it has been shown that the supercoset space $OSp(6|4)/U(3) \times SO(1, 3)$ with 24 fermionic directions, which has been used in [9]–[13] to construct a superstring sigma model in $AdS_4 \times CP^3$, is a subspace of the complete superspace and that the supercoset sigma-model action (being a partially gauge-fixed Green-Schwarz superstring action) describes only a subsector of the complete type IIA superstring theory in $AdS_4 \times CP^3$. The reason for this is that the kappa-symmetry gauge fixing condition which puts to zero eight fermionic modes corresponding to the 8 broken supersymmetries is not admissible for all possible string configurations. So, in particular, though the $OSp(6|4)/U(3) \times SO(1, 3)$ sigma model sector of the theory is classically integrable [9, 10] and there are generic arguments in favor of the integrability of the whole theory, the direct proof of the integrability of the complete $AdS_4 \times CP^3$ superstring still remains an open problem.

The knowledge of the explicit structure of the $AdS_4 \times CP^3$ superspace with 32 fermionic directions allows one to approach this and other problems. The form of the string action in the $AdS_4 \times CP^3$ superspace can be drastically simplified by choosing a suitable descrip-

tion of the background supergeometry and an appropriate kappa-symmetry gauge, as was shown previously for the cases of the type IIB superstring, D3, M2 and M5-branes in the corresponding $AdS \times S$ backgrounds [14]–[19]. A superconformal realization and a kappa-symmetry gauge fixing of the $OSp(6|4)$ sigma model sector of the $AdS_4 \times CP^3$ superstring have been considered in [20] and in a light-cone gauge in [21].

In this paper we perform an alternative κ -symmetry gauge fixing of the complete $AdS_4 \times CP^3$ superspace which is suitable for studying regions of the theory that are not reachable by the supercoset sigma model. In Subsection 4.3 we apply this gauge fixing to simplify the superstring action in $AdS_4 \times CP^3$ and consider its T-dualization along a $3d$ translationally invariant subspace of AdS_4 , similar to that performed in [17], which results in a simple action that contains fermions only up to the fourth order. We also argue that, in contrast to the $AdS_5 \times S^5$ superstring [22–24], it is not possible to T-dualize the fermionic sector of the superstring action in $AdS_4 \times CP^3$, which agrees with the conclusion of [25] regarding the $OSp(6|4)$ supercoset subsector of the theory.

In addition to the superstring, also for certain configurations of type IIA branes, e.g. D0- and D2-branes considered in section 4, the complete $AdS_4 \times CP^3$ superspace should be used. An interesting example is a $1/2$ BPS probe D2-brane placed at the $d = 3$ Minkowski boundary of AdS_4 . Upon gauge fixing worldvolume diffeomorphisms and kappa-symmetry, the effective theory on the worldvolume of this D2-brane, which describes its fluctuations in $AdS_4 \times CP^3$, is an interacting $d = 3$ gauge Born-Infeld-matter theory possessing the (spontaneously broken) superconformal symmetry $OSp(6|4)$. The model is superconformally invariant in spite of the presence on the $d = 3$ worldvolume of the dynamical Abelian vector field, since the latter is coupled to the $3d$ dilaton field associated with the radial direction of AdS_4 . The superconformal invariance is spontaneously broken by a non-zero expectation value of the dilaton. This example is a type IIA counterpart of so called singleton M2, triplet M5 and doubleton D3-branes [18, 19, 26, 27] at the boundary of $AdS_{p+2} \times S^{D-p-2}$ ($p = 2, 3$ and 5), respectively, in $D = 11$ supergravity and type IIB string theory (see [28] for a corresponding brane scan and a review of related earlier work).

Another example of interest for the study of the AdS_4/CFT_3 correspondence is a D2-brane filling $AdS_2 \times S^1 \subset AdS_4$ [29]. This BPS D2-brane configuration corresponds to a disorder loop operator in the ABJM theory. Other D-brane configurations, which are to be related to Wilson loop operators in the ABJM theory, were considered e.g. in [30]–[34]. In this paper we extend the bosonic action for a D2-brane wrapping $AdS_2 \times S^1$ to include the worldvolume fermionic modes.

We start our consideration with an overview of the geometry of the $AdS_4 \times CP^3$ superspace.

2 $AdS_4 \times CP^3$ superspace

The superspace under consideration contains $AdS_4 \times CP^3$ as its bosonic subspace and has 32 fermionic directions [4]. It is parametrized by the supercoordinates

$$Z^{\mathcal{M}} = (x^{\hat{m}}, y^{m'}, \Theta^{\alpha}) = (x^{\hat{m}}, y^{m'}, \vartheta^{\alpha\alpha'}, v^{\alpha i}), \tag{2.1}$$

where $x^{\hat{m}}$ ($\hat{m} = 0, 1, 2, 3$) and $y^{m'}$ ($m' = 1, \dots, 6$) are, respectively, the coordinates of $AdS_4 = SO(2, 3)/SO(1, 3)$ and $CP^3 = SU(4)/SU(3) \times U(1)$. Θ^α are the 32 fermionic coordinates which we split into the 24 coordinates $\vartheta^{\alpha a'}$, that correspond to the 24 unbroken supersymmetries in the $AdS_4 \times CP^3$ background, and the 8 coordinates $\nu^{\alpha i}$ which correspond to the broken supersymmetries. The indices $\alpha = 1, 2, 3, 4$ are AdS_4 spinor indices, $a' = 1, \dots, 6$ correspond to a six-dimensional representation of $SU(3)$ (note that the index a' appearing on spinors is different from the same index appearing in bosonic quantities, see appendix A.5) and $i = 1, 2$ are $SO(2) \sim U(1)$ indices. For more details of our notation and conventions see appendix A.¹ For the reader's convenience, below we list some of the notation used in the text:

1. $D = 10$ $AdS_4 \times CP^3$ superspace with 24 fermions is the supercoset $OSp(6|4)/U(3) \times SO(1, 3)$. The supervielbeins and connections are denoted by

$$\left(E^{\hat{a}}, E^{a'}, E^{\alpha a'}, \Omega^{\hat{a}\hat{b}}, \Omega^{a'b'}, \mathcal{A} \right) \tag{2.2}$$

whose expressions are given in appendix B, eq. (B.1).

2. $D = 11$ $AdS_4 \times S^7$ superspace with 24 fermions. This is obtained as a $U(1)$ bundle over the $OSp(6|4)/U(3) \times SO(1, 3)$ supercoset with the fiber coordinate denoted by z . It is the supercoset $OSp(6|4) \times U(1)/U(3) \times SO(1, 3)$ whose supervielbeins and connections are denoted by

$$\left(\hat{E}^{\hat{a}}, \hat{E}^{a'}, \hat{E}^7, \hat{E}^{\alpha a'}, \hat{\Omega}^{\hat{a}\hat{b}}, \hat{\Omega}^{a'b'} \right). \tag{2.3}$$

They are given in eqs. (4.9), see also [4]. \hat{E}^7 stands for the 7th (fiber) direction of S^7 (or, equivalently, the 11th direction in $D = 11$).

3. $D = 11$ $AdS_4 \times S^7$ superspace with 32 fermions. This is the supercoset $OSp(8|4)/SO(7) \times SO(1, 3)$. Its supervielbeins and connections are denoted by

$$\left(\underline{E}^{\hat{a}}, \underline{E}^{a'}, \underline{E}^7, \underline{E}^{\alpha a'}, \underline{E}^{\alpha i}, \underline{\Omega}^{\hat{a}\hat{b}}, \underline{\Omega}^{a'b'}, \underline{\Omega}^{a'7} \right). \tag{2.4}$$

Their explicit expressions are given in (4.1), (4.2) and (4.3).

4. Finally, the $D = 10$ $AdS_4 \times CP^3$ superspace with 32 fermionic directions is obtained by performing a rotation of (2.4) in the $(\hat{a}, 7)$ -plane accompanied by the dimensional reduction to $D = 10$ (see [4]). The geometric quantities characterizing this superspace are denoted by

$$\left(\mathcal{E}^{\hat{a}}, \mathcal{E}^{a'}, \mathcal{E}^{\alpha a'}, \mathcal{E}^{\alpha i}, \mathcal{O}^{\hat{a}\hat{b}}, \mathcal{O}^{a'b'}, \mathcal{A} \right). \tag{2.5}$$

The supervielbeins have the following form

¹Our notation and conventions are close to those in [4]. The difference is that, in this paper we put a “hat” on the AdS_4 vector indices and use a more conventional IIA superspace torsion constraint $T_{\alpha\beta}^A = -2i\Gamma_{\alpha\beta}^A$ (instead of $T_{\alpha\beta}^A = 2\Gamma_{\alpha\beta}^A$) and corresponding constraints on the gauge field strengths. We also restore the dependence of the geometric quantities and fields on the S^7 radius R , the eleven-dimensional Planck length $l_p = e^{\frac{1}{3}\langle\phi\rangle}\sqrt{\alpha'}$ and the Chern-Simons level k , which were put equal to one in [4].

$$\begin{aligned}
 \mathcal{E}^{\alpha'}(x, y, \vartheta, v) &= e^{\frac{1}{3}\phi(v)} \left(E^{\alpha'}(x, y, \vartheta) + 2iv \frac{\sinh m}{m} \gamma^{\alpha'} \gamma^5 E(x, y, \vartheta) \right), \\
 \mathcal{E}^{\hat{a}}(x, y, \vartheta, v) &= e^{\frac{1}{3}\phi(v)} \left(E^{\hat{b}}(x, y, \vartheta) + 4iv \gamma^{\hat{b}} \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} Dv \right) \Lambda_{\hat{b}}^{\hat{a}}(v) \\
 &\quad - e^{-\frac{1}{3}\phi(v)} \frac{R^2}{kl_p} \left(A(x, y, \vartheta) - \frac{4}{R} v \varepsilon \gamma^5 \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} Dv \right) E_{7\hat{a}}(v), \\
 \mathcal{E}^{\alpha i}(x, y, \vartheta, v) &= e^{\frac{1}{6}\phi(v)} \left(\frac{\sinh \mathcal{M}}{\mathcal{M}} Dv \right)^{\beta j} S_{\beta j}^{\alpha i}(v) - i e^{\phi(v)} \mathcal{A}_1(x, y, \vartheta, v) (\gamma^5 \varepsilon \lambda(v))^{\alpha i}, \\
 \mathcal{E}^{\alpha \alpha'}(x, y, \vartheta, v) &= e^{\frac{1}{6}\phi(v)} E^{\gamma b'}(x, y, \vartheta) \left(\delta_{\gamma}^{\beta} - \frac{8}{R} \left(\gamma^5 v \frac{\sinh^2 m/2}{m^2} \right)_{\gamma i} v^{\beta i} \right) S_{\beta b'}^{\alpha \alpha'}(v).
 \end{aligned} \tag{2.6}$$

The new objects appearing in these expressions, m , \mathcal{M} , $\Lambda_{\hat{a}}^{\hat{b}}$, $E_{7\hat{a}}$ and $S_{\underline{\alpha}}^{\beta}$, are functions of v and their explicit forms are given in appendix B.1 while the dilaton ϕ , dilatino λ and RR one-form \mathcal{A}_1 are given below. Contracted spinor indices have been suppressed, e.g. $(v \varepsilon \gamma^5)_{\alpha i} = v^{\beta j} \varepsilon_{ji} \gamma_{\beta \alpha}^5$, where $\varepsilon_{ij} = -\varepsilon_{ji}$, $\varepsilon_{12} = 1$ is the SO(2) invariant tensor. The covariant derivative is defined as

$$Dv = \left(d + \frac{i}{R} E^{\hat{a}}(x, y, \vartheta) \gamma^5 \gamma_{\hat{a}} - \frac{1}{4} \Omega^{\hat{a}\hat{b}}(x, y, \vartheta) \gamma_{\hat{a}\hat{b}} \right) v. \tag{2.7}$$

The type IIA RR one-form gauge superfield is

$$\begin{aligned}
 \mathcal{A}_1(x, y, \vartheta, v) &= R e^{-\frac{4}{3}\phi(v)} \left[\left(A(x, y, \vartheta) - \frac{4}{R} v \varepsilon \gamma^5 \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} Dv \right) \frac{R}{kl_p} \Phi(v) \right. \\
 &\quad \left. + \frac{1}{kl_p} \left(E^{\hat{a}}(x, y, \vartheta) + 4iv \gamma^{\hat{a}} \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} Dv \right) E_{7\hat{a}}(v) \right].
 \end{aligned} \tag{2.8}$$

The RR four-form and the NS-NS three-form superfield strengths are given by

$$\begin{aligned}
 F_4 &= d\mathcal{A}_3 - \mathcal{A}_1 H_3 = -\frac{1}{4!} \mathcal{E}^{\hat{d}} \mathcal{E}^{\hat{c}} \mathcal{E}^{\hat{b}} \mathcal{E}^{\hat{a}} \left(\frac{6}{kl_p} e^{-2\phi} \Phi \varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}} \right) - \frac{i}{2} \mathcal{E}^B \mathcal{E}^A \mathcal{E}^{\beta} \mathcal{E}^{\alpha} e^{-\phi} (\Gamma_{AB})_{\underline{\alpha}\beta}, \\
 H_3 &= dB_2 = -\frac{1}{3!} \mathcal{E}^{\hat{c}} \mathcal{E}^{\hat{b}} \mathcal{E}^{\hat{a}} \left(\frac{6}{kl_p} e^{-\phi} \varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}} E_{7\hat{d}} \right) - i \mathcal{E}^A \mathcal{E}^B \mathcal{E}^{\beta} \mathcal{E}^{\alpha} (\Gamma_A \Gamma_{11})_{\underline{\alpha}\beta} + i \mathcal{E}^B \mathcal{E}^A \mathcal{E}^{\alpha} (\Gamma_{AB} \Gamma^{11} \lambda)_{\underline{\alpha}}
 \end{aligned} \tag{2.9}$$

and the corresponding gauge potentials are

$$B_2 = b_2 + \int_0^1 dt i_{\Theta} H_3(x, y, t\Theta), \quad \Theta = (\vartheta, v) \tag{2.10}$$

$$\mathcal{A}_3 = a_3 + \int_0^1 dt i_{\Theta} (F_4 + \mathcal{A}_1 H_3)(x, y, t\Theta), \tag{2.11}$$

where b_2 and a_3 are the purely bosonic parts of the gauge potentials and i_{Θ} means the inner product with $\Theta^{\underline{\alpha}}$. Note that b_2 is pure gauge in the $AdS_4 \times CP^3$ solution while a_3 is the RR three-form potential of the bosonic background.

The dilaton superfield $\phi(v)$, which depends only on the eight fermionic coordinates corresponding to the broken supersymmetries, has the following form in terms of $E_{7\hat{a}}(v)$

and $\Phi(v)$

$$e^{\frac{2}{3}\phi(v)} = \frac{R}{kl_p} \sqrt{\Phi^2 + E_7^{\hat{a}} E_7^{\hat{b}} \eta_{\hat{a}\hat{b}}}. \quad (2.12)$$

The value of the dilaton at $v = 0$ is

$$e^{\frac{2}{3}\phi(v)}|_{v=0} = e^{\frac{2}{3}\phi_0} = \frac{R}{kl_p}. \quad (2.13)$$

The fermionic field $\lambda^{\alpha i}(v)$ describes the non-zero components of the dilatino superfield and is given by the equation [35]

$$\lambda_{\alpha i} = -\frac{i}{3} D_{\alpha i} \phi(v). \quad (2.14)$$

In the above expressions $E^{\hat{a}}(x, y, \vartheta)$, $E^{a'}(x, y, \vartheta)$ and $\Omega^{\hat{a}\hat{b}}(x, y, \vartheta)$ are the supervielbeins and the AdS_4 part of the spin connection of the supercoset $O\text{Sp}(6|4)/U(3) \times \text{SO}(1, 3)$ and $A(x, y, \vartheta)$ is the corresponding type IIA RR one-form gauge superfield, eq. (2.2), whose explicit form is given in appendix B.

As mentioned above other quantities appearing in eqs. (2.6)–(2.14), namely \mathcal{M} , m , $\Phi(v)$, $E_7^{\hat{a}}(v)$, $\Lambda_{\hat{a}}^{\hat{b}}(v)$ and $S_{\underline{\beta}}^{\underline{\alpha}}(v)$, whose geometrical and group-theoretical meaning has been explained in [4], are also given in appendix B.

3 Kappa-symmetry gauge fixing

We shall now consider conditions for gauge fixing kappa-symmetry which are convenient for the description of configurations of superstrings and D-branes in the $AdS_4 \times CP^3$ superbackground described above and for studying AdS_4/CFT_3 correspondence problems.

Since the AdS_4/CFT_3 holography is realized at the 3d Minkowski boundary of AdS_4 it is convenient to choose the $AdS_4 \times CP^3$ metric in the form

$$ds^2 = \left(\frac{r}{R_{CP^3}}\right)^4 dx^m \eta_{mn} dx^n + \left(\frac{R_{CP^3}}{r}\right)^2 dr^2 + R_{CP^3}^2 ds_{CP^3}^2 \quad (3.1)$$

where $m = 0, 1, 2$ are indices corresponding to the coordinates of the 3d Minkowski boundary and r is the 4th, radial, coordinate of AdS_4 . So the AdS_4 coordinates are $x^{\hat{m}} = (x^m, r)$. The AdS_4 radius is half of the CP^3 radius R_{CP^3} which (in the string frame) is related to the S^7 radius R as follows

$$R_{CP^3} = e^{\frac{1}{3}\phi_0} R = \left(\frac{R^3}{kl_p}\right)^{1/2}. \quad (3.2)$$

In the coordinate system associated with the metric (3.1) (the bosonic part of) the RR field \mathcal{A}_3 , whose flux, together with $F_2 = da_1 = \frac{e^{-\phi_0}}{R_{CP^3}} dy^{m'} dy^{n'} J_{m'n'}$ (where $dy^{m'} dy^{n'} J_{m'n'}$ is the Kähler form on CP^3), ensures the compactification on $AdS_4 \times CP^3$ [6–8], has the following form

$$a_3 = e^{-\phi_0} \left(\frac{r}{R_{CP^3}}\right)^6 dx^0 dx^1 dx^2, \quad F_4 = \frac{6}{R_{CP^3}} e^{-\phi_0} \left(\frac{r}{R_{CP^3}}\right)^5 dx^0 dx^1 dx^2 dr. \quad (3.3)$$

(In our conventions the exterior derivative acts from the right.)

Instead of the AdS_4 part of the metric (3.1), which obscures a bit the fact that the metric of the conformal boundary is the flat Minkowski metric on $R^{1,2}$, one can use the AdS_4 metric in the conformally flat form

$$ds_{AdS_4}^2 = \frac{1}{u^2} \left(dx^m \eta_{mn} dx^n + \frac{R_{CP^3}^2}{4} du^2 \right), \quad u = \left(\frac{R_{CP^3}}{r} \right)^2. \quad (3.4)$$

This metric is associated with a simple coset representative $g = \exp(x^m \Pi_m) \exp(R_{CP^3} \ln(u) D)$, where Π_m are the generators of the Poincaré translations along the Minkowski boundary ($[\Pi_m, \Pi_n] = 0$) and D is the dilatation generator $[D, \Pi_m] = \Pi_m$.

Note that if the components of the vielbein associated with the metric (3.1) or (3.4) are chosen to be²

$$e^{\frac{\phi_0}{3}} e^a = \frac{r^2}{R_{CP^3}^2} dx^a = u^{-1} dx^a, \quad e^{\frac{\phi_0}{3}} e^3 = \frac{R_{CP^3}}{r} dr = -\frac{R_{CP^3}}{2u} du, \quad (3.5)$$

the components of the $SO(1,3)$ spin connection are

$$\omega^{a3} = -\frac{2}{R} e^a, \quad (3.6)$$

and

$$\omega^{ab} = 0. \quad (3.7)$$

We shall use the relation (3.6) to simplify the form of the gauge fixed $AdS_4 \times CP^3$ supergeometry. Note that the condition (3.6) can always be imposed by performing an appropriate local $SO(1,3)$ transformations of the vielbein and connection, though in general the $SO(1,2)$ components ω^{ab} of the connection will be non-zero.

Using the previous experience of gauge fixing kappa-symmetry of superstrings, D-branes and M-branes in AdS backgrounds [14]–[19] we choose the kappa-symmetry gauge fixing condition in the form³

$$\Theta = \frac{1}{2}(1 \pm \gamma)\Theta \quad \Rightarrow \quad \vartheta^{a'} = \frac{1}{2}(1 \pm \gamma)\vartheta^{a'}, \quad v^i = \frac{1}{2}(1 \pm \gamma)v^i, \quad (3.8)$$

where

$$\gamma = \gamma^{012} \quad \Rightarrow \quad \gamma^2 = 1, \quad \{\gamma, \gamma^5\} = [\gamma, \gamma^a] = 0 \quad \text{and} \quad \gamma\gamma^3 = -i\gamma^5, \quad (3.9)$$

$a = 0, 1, 2$ are the indices of the 3d Minkowski boundary or of $AdS_2 \times S^1$ and γ^3 is associated with the third spatial direction of AdS_4 . Note that, in view of our definition (A.10) of the $D = 10$ gamma-matrices the matrices defined in (3.9) can be regarded either as $4d$ gamma matrices or as the $D = 10$ matrices $\Gamma^{\hat{a}} = \gamma^{\hat{a}} \otimes \mathbf{1}$ ($\hat{a} = 0, 1, 2, 3$).

²Note that the vielbeins e^a and e^3 appearing in eq. (3.5) correspond to the AdS_4 metric of the $D = 11$ $AdS_4 \times S^7$ solution characterized by the radius R which is related to the CP^3 radius in the string frame according to eq. (3.2). These bosonic vielbeins will appear in our explicit expressions for the $AdS_4 \times CP^3$ supergeometry.

³Such a gauge for fixing kappa-symmetry is analogous to the so called Killing spinor gauge [14], or supersolvable gauge [36], or the superconformal gauge [19].

The condition (3.8) is admissible for fixing kappa-symmetry if the projection matrix $\frac{1}{2}(1 \mp \gamma)$ either coincides or does not commute with the kappa-symmetry projection matrix $\frac{1}{2}(1 + \Gamma)$ of a given configuration of the superstring and D-branes. This can be understood in the following way. To lowest order in fermions Θ transforms under kappa-symmetry as

$$\delta_\kappa \Theta = \frac{1}{2}(1 + \Gamma)\kappa, \tag{3.10}$$

where $\frac{1}{2}(1 + \Gamma)$ is a projection matrix and $\kappa(\xi)$ is an arbitrary spinor parameter. It is then clear that if the two projectors coincide, we can pick a κ such that $\frac{1}{2}(1 + \Gamma)\Theta = 0$, or equivalently $\Theta = \frac{1}{2}(1 - \Gamma)\Theta$. In the case when the two projection operators do not coincide a kappa-symmetry variation of the gauge-fixing condition $\frac{1}{2}(1 \mp \gamma)\Theta = 0$ which leaves it intact gives

$$0 = \frac{1}{4}(1 \mp \gamma)(1 + \Gamma)\kappa = \frac{1}{8}(1 + \Gamma)(1 \mp \gamma)(1 + \Gamma)\kappa \mp \frac{1}{8}[\gamma, \Gamma](1 + \Gamma)\kappa = \mp \frac{1}{8}[\gamma, \Gamma](1 + \Gamma)\kappa, \tag{3.11}$$

where in the last step we made use of the initial equation. This means that for the gauge-fixing to be complete, i.e. that the variation of the gauge fixing condition vanishes if and only if all independent kappa-symmetry parameters are put to zero, the commutator $[\gamma, \Gamma]$ has to be an invertible matrix (when restricted to the relevant subspace).

As we shall see below, for any choice of the sign the condition (3.8) is an admissible gauge-fixing in the case of arbitrary motion of D0-branes in $AdS_4 \times CP^3$, while in the case of the superstring it is admissible (for both signs) for those configurations for which the projection of the string worldsheet on the 3d Minkowski boundary is a non-degenerate two-dimensional time-like surface. In the case of the D2-brane placed at the Minkowski boundary of AdS_4 , to gauge fix kappa-symmetry one must choose the condition (3.8) with the lower sign [19], while both signs are admissible when the D2-brane wraps an $AdS_2 \times S^1$ subspace of AdS_4 . However, the choice of (3.8) with the upper sign yields the simplest gauge-fixed form of the string and brane actions in the $AdS_4 \times CP^3$ superbackground.

When the fermionic coordinates are restricted by the condition (3.8), the expressions for the supervielbeins and the gauge superfields of the $AdS_4 \times CP^3$ superspace drastically simplify due to the identities satisfied by the projected fermionic coordinates given in appendix C. In particular, the functions of v which enter the eqs. (2.6)–(2.12), whose explicit forms are given in appendix B.1, reduce to

$$\Phi(v) = 1 + \frac{8}{R} v \varepsilon \gamma^5 \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \varepsilon v = 1, \tag{3.12}$$

$$E_7^a(v) = -\frac{8i}{R} v \gamma^a \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \varepsilon v = -\frac{2i}{R} v \gamma^a \varepsilon v, \tag{3.13}$$

$$E_7^3(v) = -\frac{8i}{R} v \gamma^3 \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \varepsilon v = 0. \tag{3.14}$$

The dilaton superfield (2.12) takes the form

$$e^{\frac{2}{3}\phi(v)} = \frac{R}{kl_p} \left(1 - \frac{6}{R^2} (vv)^2 \right) \Rightarrow \phi(v) = \frac{3}{2} \left(\log \frac{R}{kl_p} - \frac{6}{R^2} (vv)^2 \right), \tag{3.15}$$

where $\nu\nu = \delta_{ij}C_{\alpha\beta}v^{\alpha i}v^{\beta j}$, and the dilatino becomes

$$\lambda^{\alpha i} = \frac{2i}{R} \left(\frac{R}{kl_p} \right)^{-1/4} \left((\gamma^5 v)^{\alpha i} + \frac{3}{R} v^{\alpha i} \nu\nu \right). \quad (3.16)$$

We also find that

$$\Lambda_a{}^b = \left(1 + \frac{2}{R^2} (\nu\nu)^2 \right) \delta_a{}^b, \quad \Lambda_3{}^a = \Lambda_a{}^3 = 0, \quad \Lambda_3{}^3 = 1, \quad (3.17)$$

and

$$S_{\underline{\alpha}\underline{\beta}} = \left(\frac{R}{kl_p} \right)^{1/4} e^{-\frac{1}{6}\phi} \delta_{\underline{\alpha}\underline{\beta}} + \frac{i}{R} v\gamma^a \varepsilon\nu (\Gamma_a \Gamma_{11})_{\underline{\alpha}\underline{\beta}}. \quad (3.18)$$

3.1 $AdS_4 \times CP^3$ supergeometry with $\Theta = \frac{1}{2}(1 + \gamma)\Theta$

The supervielbeins (2.6) and the gauge superfields (2.8), (2.10) and (2.11) take the simplest form when the kappa-symmetry gauge condition (3.8) is chosen with the upper sign. In virtue of eqs. (3.12)–(3.18) and expressions given in appendix C, the supervielbeins reduce to

$$\begin{aligned} \mathcal{E}^{a'}(x, y, \vartheta, \nu) &= \left(\frac{R}{kl_p} \right)^{1/2} e^{a'}(y) \left(1 - \frac{3}{R^2} (\nu\nu)^2 \right), \\ \mathcal{E}^a(x, y, \vartheta, \nu) &= \left(\frac{R}{kl_p} \right)^{1/2} \left(e^a(x) + i\Theta\gamma^a D\Theta \right) \left(1 - \frac{1}{R^2} (\nu\nu)^2 \right), \\ \mathcal{E}^3(x, y, \vartheta, \nu) &= \left(\frac{R}{kl_p} \right)^{1/2} e^3(x) \left(1 - \frac{3}{R^2} (\nu\nu)^2 \right), \\ \mathcal{E}^{\alpha i}(x, y, \vartheta, \nu) &= \left(\frac{R}{kl_p} \right)^{1/4} \left((D_8 v)^{\alpha i} - \frac{1}{R} v\gamma^a \varepsilon\nu (D_8 v \varepsilon \gamma_a \gamma_5)^{\alpha i} \right. \\ &\quad \left. - \frac{4i}{R^2} (e^a(x) + i\Theta\gamma^a D\Theta) (\gamma_a v)^{\alpha i} \nu\nu \right), \\ \mathcal{E}^{\alpha a'}(x, y, \vartheta, \nu) &= \left(\frac{R}{kl_p} \right)^{1/4} \left((D_{24} \vartheta)^{\alpha a'} + \frac{i}{R} v\gamma^a \varepsilon\nu (D_{24} \vartheta \gamma_a \gamma_5 \gamma_7)^{\alpha a'} \right). \end{aligned} \quad (3.19)$$

The type IIA RR one-form gauge superfield is

$$\mathcal{A}_1(x, y, \vartheta, \nu) = kl_p \left(A(y) - \frac{2i}{R^2} (e^a(x) + i\Theta\gamma^a D\Theta) \nu \varepsilon \gamma_a \nu \right), \quad (3.20)$$

where $A(y)$ is the potential for the Kähler form on CP^3 , i.e. $dA(y) = \frac{1}{R^2} dy^{m'} dy^{n'} J_{m'n'}$, and the covariant derivatives are

$$\begin{aligned} D\Theta &= (D_8 v, D_{24} \vartheta) \\ D_8 v &= \mathcal{P}_2 \left(d - \frac{1}{R} e^3 - \frac{1}{4} \omega^{ab} \gamma_{ab} + 2A(y) \varepsilon \right) v \\ D_{24} \vartheta &= \mathcal{P}_6 \left(d - \frac{1}{R} e^3 + \frac{i}{R} e^{a'} \gamma_{a'} - \frac{1}{4} \omega^{ab} \gamma_{ab} - \frac{1}{4} \omega^{a'b'} \gamma_{a'b'} \right) \vartheta, \end{aligned} \quad (3.21)$$

where \mathcal{P}_2 and \mathcal{P}_6 are projectors that single out from 32 Θ , respectively, 8 v and 24 ϑ (see appendix A.5). The appearance of the U(1) gauge potential $A(y)$ in the covariant derivative of v (3.21) reflects the fact that v has U(1) charge equal to 2.

Note that

$$D_8 = \mathcal{P}_2 \mathcal{D} \mathcal{P}_2, \quad D_{24} = \mathcal{P}_6 \mathcal{D} \mathcal{P}_6, \quad (3.22)$$

where

$$\mathcal{D} = d - \frac{1}{R} e^3 + \frac{i}{R} e^{a'} \gamma_{a'} - \frac{1}{4} \omega^{ab} \gamma_{ab} - \frac{1}{4} \omega^{a'b'} \gamma_{a'b'}. \quad (3.23)$$

The NS-NS three-form, eq. (2.9), becomes

$$\begin{aligned} H_3 = & -\frac{6i}{R^2} e^3 \mathcal{E}^b \mathcal{E}^a \varepsilon_{abc} v \gamma^c \varepsilon v - \frac{2R}{kl_p} \left[\frac{i}{R} (e^b + i\Theta \gamma^b D\Theta) (e^a + i\Theta \gamma^a D\Theta) D_8 v \gamma_{ab} \varepsilon v \right. \\ & + \frac{1}{R} (e^a + i\Theta \gamma^a D\Theta) D\Theta \gamma_{ab} D\Theta v \gamma^b \varepsilon v + \frac{1}{2} e^3 D\Theta \gamma_{\tau} D\Theta + \frac{2i}{R} e^3 e^{a'} D\Theta \gamma_{a'} \gamma_{\tau} v \\ & \left. + \frac{i}{2} e^{a'} D\Theta \gamma_{a'} \gamma_{\tau} D\Theta + \frac{1}{R} e^{b'} e^{a'} D\Theta \gamma_{a'b'} \gamma_{\tau} v \right], \end{aligned} \quad (3.24)$$

where $\Theta = (\vartheta, v)$ and $D\Theta = (D_{24}\vartheta, D_8 v)$.

We now want to determine the potential of $H_3 = dB_2$ using eq. (2.10). Taking into account that $i_{\Theta} \mathcal{E}^A = 0$, and the fact that with the plus sign in the projector (3.8) $\Theta \gamma_{ab} D\Theta = \varepsilon_{abc} \Theta \gamma^c D\Theta$ etc., we get

$$\begin{aligned} i_{\Theta} H_3 = & 2 \frac{R}{kl_p} \left(-\frac{i}{R} e^b e^a v \gamma_{ab} \varepsilon v - \frac{2i}{R} e^3 e^{a'} \vartheta \gamma_{a'} \gamma^{\tau} v - \frac{1}{R} e^{b'} e^{a'} \Theta \gamma_{a'b'} \gamma^{\tau} v + e^3 \Theta \gamma^{\tau} D\Theta \right. \\ & \left. + i e^{a'} \Theta \gamma_{a'} \gamma^{\tau} D\Theta + \frac{4}{R} e^b \Theta \gamma^a D\Theta v \gamma_{ab} \varepsilon v + \frac{3i}{R} \Theta \gamma^b D\Theta \Theta \gamma^a D\Theta v \gamma_{ab} \varepsilon v \right). \end{aligned} \quad (3.25)$$

This gives the NS-NS two-form potential (see eq. (2.10))

$$\begin{aligned} B_2 = & \frac{R}{kl_p} \left[-\frac{i}{R} (e^b + i\Theta \gamma^b D\Theta) (e^a + i\Theta \gamma^a D\Theta) v \gamma_{ab} \varepsilon v - \frac{2i}{R} e^3 e^{a'} \vartheta \gamma_{a'} \gamma^{\tau} v \right. \\ & \left. - \frac{1}{R} e^{b'} e^{a'} \Theta \gamma_{a'b'} \gamma^{\tau} v + e^3 \Theta \gamma^{\tau} D\Theta + i e^{a'} \Theta \gamma_{a'} \gamma^{\tau} D\Theta \right]. \end{aligned} \quad (3.26)$$

Now we turn our attention to the RR four-form F_4 (2.9) and its potential \mathcal{A}_3 (2.11). F_4 simplifies to

$$F_4 = -\frac{1}{kl_p} e^{-2\phi} \mathcal{E}^3 \mathcal{E}^c \mathcal{E}^b \mathcal{E}^a \varepsilon_{abc} - \frac{i}{2} e^{-\phi} \mathcal{E}^B \mathcal{E}^A \mathcal{E}^{\beta} \mathcal{E}^{\alpha} (\Gamma_{AB})_{\alpha\beta}, \quad (3.27)$$

which gives

$$\begin{aligned} i_{\Theta} F_4 = & -i \left(\frac{R}{kl_p} \right)^{1/4} e^{-\phi} \mathcal{E}^B \mathcal{E}^A \left(\mathcal{E} \Gamma_{AB} \Theta + \frac{i}{R} \mathcal{E} \Gamma_{AB} \gamma_a \Gamma_{11} \Theta v \gamma^a \varepsilon v \right) \\ = & -i (e^b + i\Theta \gamma^b D\Theta) (e^a + i\Theta \gamma^a D\Theta) \left(1 + \frac{12}{R^2} (vv)^2 \right) \left(D\Theta \gamma_{ab} \Theta + \frac{4i}{R^2} e^c \varepsilon_{abc} (vv)^2 \right) \\ & + \frac{4i}{R} e^3 (e^a + i\Theta \gamma^a D\Theta) D\Theta \gamma^{\tau} \Theta v \gamma_a \varepsilon v - \frac{4}{R} e^{a'} (e^a + i\Theta \gamma^a D\Theta) D\Theta \gamma_{a'} \gamma^{\tau} \Theta v \gamma_a \varepsilon v \end{aligned}$$

$$\begin{aligned}
& + 2e^3 e^{a'} \left(D\Theta \gamma_{a'} \Theta + \frac{4i}{R^2} (e^a + i\Theta \gamma^a D\Theta) v \gamma_a \gamma_{a'} \vartheta v v \right) \\
& - i e^{b'} e^{a'} \left(D\Theta \gamma_{a'b'} \Theta + \frac{4i}{R^2} (e^a + i\Theta \gamma^a D\Theta) v \gamma_a \gamma_{a'b'} \vartheta v v \right). \tag{3.28}
\end{aligned}$$

Since $i_{\Theta} \mathcal{A}_1 = 0$ the RR three-form potential (2.11) becomes

$$\begin{aligned}
\mathcal{A}_3 &= a_3 + \int_0^1 dt (i_{\Theta} F_4 + \mathcal{A}_1 i_{\Theta} H_3)(x, y, t\Theta) \\
&= a_3 - \frac{i}{2} e^b e^a D\Theta \gamma_{ab} \Theta + e^3 e^{a'} D\Theta \gamma_{a'} \Theta - \frac{i}{2} e^{b'} e^{a'} D\Theta \gamma_{a'b'} \Theta + \frac{1}{2} e^b \Theta \gamma^a D\Theta D\Theta \gamma_{ab} \Theta \\
&\quad + \frac{i}{6} \Theta \gamma^b D\Theta \Theta \gamma^a D\Theta D\Theta \gamma_{ab} \Theta + k l_p A(y) B_2. \tag{3.29}
\end{aligned}$$

Looking at the purely bosonic part of F_4 , eq. (3.27) it is easy to see (compare also with eqs. (3.3)) that we can take

$$a_3 = \frac{1}{3!} e^c e^b e^a \varepsilon_{abc}. \tag{3.30}$$

Note that in the above expressions for the supervielbeins (3.19), the RR one-form (3.20), the three-form (3.29) and the NS-NS two-form (3.26) the maximum order of the fermions is six.

3.2 $AdS_4 \times CP^3$ supergeometry with $\Theta = \frac{1}{2}(1 - \gamma)\Theta$

When the condition (3.8) is chosen with the lower sign, in view of eqs. (3.12)–(3.18) and expressions given in appendix C, the supervielbeins (2.6) and the RR one-form gauge superfield (2.8) reduce to a form which is more complicated than their gauge-fixed counterparts of the previous Subsection. But, as we have already mentioned, one cannot use the gauge fixing condition of Subsection 3.1 to describe the D2-brane at the Minkowski boundary of AdS_4 , and should impose $\Theta = \frac{1}{2}(1 - \gamma)\Theta$ instead. In this case the supervielbeins take the following form

$$\begin{aligned}
\mathcal{E}^{a'}(x, y, \vartheta, v) &= \left(\frac{R}{kl_p} \right)^{1/2} \left(e^{a'}(y) - \frac{2}{R} e^a(x) \Theta \gamma^{a'} \gamma_a \Theta \right) \left(1 - \frac{3}{R^2} (vv)^2 \right), \\
\mathcal{E}^a(x, y, \vartheta, v) &= \left(\frac{R}{kl_p} \right)^{1/2} \left(e^a(x) + i\Theta \gamma^a D\Theta + \frac{1}{R^2} e^a(x) (\vartheta\vartheta - vv)^2 \right) \\
&\quad \times \left(1 - \frac{1}{R^2} (vv)^2 \right), \\
\mathcal{E}^3(x, y, \vartheta, v) &= \left(\frac{R}{kl_p} \right)^{1/2} e^3(x) \left(1 - \frac{3}{R^2} (vv)^2 \right), \tag{3.31} \\
\mathcal{E}^{\alpha i}(x, y, \vartheta, v) &= \left(\frac{R}{kl_p} \right)^{1/4} \left((D_8 v)^{\alpha i} - \frac{1}{R} v \gamma^a \varepsilon v (D_8 v \varepsilon \gamma_a \gamma_5)^{\alpha i} \right. \\
&\quad \left. - \frac{4i}{R^2} (e^a(x) + i\Theta \gamma^a D\Theta + \frac{1}{R^2} e^a(x) (\vartheta\vartheta - vv)^2) (\gamma_a v)^{\alpha i} v v \right), \\
\mathcal{E}^{\alpha a'}(x, y, \vartheta, v) &= \left(\frac{R}{kl_p} \right)^{1/4} \left((D_{24} \vartheta)^{\alpha a'} + \frac{i}{R} (D_{24} \vartheta \gamma_a \gamma^5 \gamma^7)^{\alpha a'} v \gamma^a \varepsilon v \right).
\end{aligned}$$

The type IIA RR one-form gauge superfield is

$$\mathcal{A}_1(x, y, \vartheta, v) = kl_p \left(A(y) - \frac{2}{R^2} e^a(x) \Theta \gamma^7 \gamma_a \Theta - \frac{2i}{R^2} (e^a(x) + i\Theta \gamma^a D\Theta + \frac{1}{R^2} e^a(x) (\vartheta\vartheta)^2) v \varepsilon \gamma_a v \right). \quad (3.32)$$

In the above expressions

$$\begin{aligned} D\Theta &= (D_8 v, D_{24} \vartheta), \\ D_8 v &= \left(D - \frac{2i}{R^2} v v e^a \gamma_a + 2A(x, y, \vartheta) \varepsilon \right) v \\ &= \left(d + \frac{2i}{R} e^a (\gamma^5 \gamma_a + \frac{1}{R} (\vartheta\vartheta - v v) \gamma_a) + \frac{1}{R} e^3 \right. \\ &\quad \left. - \frac{1}{4} \omega^{ab} \gamma_{ab} + (2A(y) - \frac{4}{R^2} e^a \vartheta \gamma^7 \gamma_a \vartheta) \varepsilon \right) v, \end{aligned} \quad (3.33)$$

$$\begin{aligned} D_{24} \vartheta &= \mathcal{P}_6 \left(d + \frac{2i}{R} e^a (\gamma^5 \gamma_a + \frac{1}{R} (\vartheta\vartheta - v v) \gamma_a) + \frac{1}{R} e^3 \right. \\ &\quad \left. + \frac{i}{R} e^{a'} \gamma_{a'} - \frac{1}{4} \omega^{ab} \gamma_{ab} - \frac{1}{4} \omega^{a'b'} \gamma_{a'b'} \right) \vartheta. \end{aligned} \quad (3.34)$$

(The shift of D by $-\frac{2i}{R^2} v v e^a \gamma_a$ has been made for the expressions to have a nicer and more covariant-looking form).

The NS-NS three-form, eq. (2.9), becomes

$$H_3 = -\frac{6i}{R^2} e^3 \mathcal{E}^b \mathcal{E}^a \varepsilon_{abc} v \gamma^c \varepsilon v - i \mathcal{E}^A \mathcal{E}^\beta \mathcal{E}^\alpha (\Gamma_A \Gamma_{11})_{\alpha\beta} + i \mathcal{E}^B \mathcal{E}^A \mathcal{E}^\alpha (\Gamma_{AB} \Gamma^{11} \lambda)_{\underline{\alpha}}. \quad (3.35)$$

We now would like to determine its potential according to eq. (2.10). Using the fact that

$$i_\Theta \mathcal{E}^\alpha = \left(\frac{R}{kl_p} \right)^{1/4} (\Theta^\alpha + \frac{i}{R} v \gamma^a \varepsilon v (\Theta \Gamma_a \Gamma_{11})^\alpha) \quad (3.36)$$

and $i_\Theta \mathcal{E}^A = 0$ we get

$$\begin{aligned} i_\Theta H_3 &= \frac{R}{kl_p} \left(-\frac{2}{R} \left(e^b + i\Theta \gamma^b D\Theta + \frac{1}{R^2} e^b (\vartheta\vartheta - v v)^2 \right) \left(e^a + i\Theta \gamma^a D\Theta + \frac{1}{R^2} e^a (\vartheta\vartheta - v v)^2 \right) v \gamma_{ab} \gamma^7 v \right. \\ &\quad + \frac{4}{R} \left(e^a + i\Theta \gamma^a D\Theta + \frac{1}{R^2} e^a (\vartheta\vartheta - v v)^2 \right) D\Theta \gamma_{ba} \Theta v \gamma^b \varepsilon v + \frac{8}{R^2} e^3 e^a \vartheta\vartheta v \gamma_a \varepsilon v \\ &\quad + \frac{4}{R} \left(e^a + i\Theta \gamma^a D\Theta + \frac{1}{R^2} e^a (\vartheta\vartheta - v v)^2 \right) e^b \Theta \gamma_{ba} \gamma^7 \Theta + 2e^3 D\Theta \gamma^7 \Theta \\ &\quad - 2i \left(e^{a'} - \frac{2}{R} e^a \Theta \gamma^{a'} \gamma_a \Theta \right) D\Theta \gamma_{a'} \gamma^7 \Theta + \frac{4i}{R} e^3 \left(e^{a'} - \frac{2}{R} e^c \Theta \gamma^{a'} \gamma_c \Theta \right) \vartheta \gamma_{a'} \gamma^7 v \\ &\quad - \frac{2}{R} \left(e^{b'} - \frac{2}{R} e^b \Theta \gamma^{b'} \gamma_b \Theta \right) \left(e^{a'} - \frac{2}{R} e^c \Theta \gamma^{a'} \gamma_c \Theta \right) \Theta \gamma_{a'b'} \gamma^7 v \\ &\quad \left. + \frac{8i}{R^2} \left(e^{a'} - \frac{2}{R} e^c \Theta \gamma^{a'} \gamma_c \Theta \right) e^a \Theta \gamma_{a'} \gamma_{ab} \Theta v \gamma^b \varepsilon v \right) \end{aligned} \quad (3.37)$$

and finally

$$\begin{aligned}
 B_2 = & \frac{R}{kl_p} \left(\frac{i}{R} \left(e^b + i\Theta\gamma^b D\Theta + \frac{1}{R^2} e^b (\vartheta\vartheta)^2 \right) \left(e^a + i\Theta\gamma^a D\Theta + \frac{1}{R^2} e^a (\vartheta\vartheta)^2 \right) \varepsilon_{abc} v\gamma^c \varepsilon v \right. \\
 & + \frac{2}{R} \left(e^a + \frac{i}{2} \Theta\gamma^a D\Theta + \frac{1}{3R^2} e^a (\vartheta\vartheta - vv)^2 \right) e^b \varepsilon_{abc} \Theta\gamma^c \gamma^7 \Theta \\
 & + \frac{2i}{R} e^3 \left(e^{a'} - \frac{1}{R} e^a \Theta\gamma^{a'} \gamma_a \Theta \right) \vartheta\gamma_{a'} \gamma^7 v \\
 & - \frac{1}{R} \left(e^{b'} - \frac{1}{R} e^b \Theta\gamma^{b'} \gamma_b \Theta \right) \left(e^{a'} - \frac{1}{R} e^a \Theta\gamma^{a'} \gamma_a \Theta \right) \Theta\gamma_{a'b'} \gamma^7 v \\
 & - \frac{1}{3R^3} e^b \Theta\gamma^{b'} \gamma_b \Theta e^c \Theta\gamma^{a'} \gamma_c \Theta \Theta\gamma_{a'b'} \gamma^7 v \\
 & - e^3 \Theta\gamma^7 D\Theta + \frac{i}{R^2} e^3 e^a \left(\vartheta\gamma^7 \gamma_a \vartheta - v\gamma_a \gamma^7 v \right) \Theta\Theta \\
 & + i \left(e^{a'} - \frac{1}{R} e^a \Theta\gamma^{a'} \gamma_a \Theta \right) \Theta\gamma_{a'} \gamma^7 D\Theta \\
 & + \frac{2i}{R^2} \left(e^{a'} - \frac{4}{3R} e^c \Theta\gamma^{a'} \gamma_c \Theta \right) e^a \varepsilon_{abc} \Theta\gamma_{a'} \gamma^b \Theta v\gamma^c \varepsilon v \\
 & + \frac{2}{R^2} \left(e^{a'} - \frac{2}{3R} e^b \Theta\gamma^{a'} \gamma_b \Theta \right) e^a \vartheta\gamma^7 \gamma_a \vartheta \vartheta\gamma_{a'} v \\
 & + \frac{1}{R^2} \left(e^{a'} - \frac{2}{3R} e^b \Theta\gamma^{a'} \gamma_b \Theta \right) e^a \Theta\gamma_{a'} \gamma^7 \gamma_a \Theta (\vartheta\vartheta - vv) \\
 & - \frac{2}{3R^3} e^b e^a \varepsilon_{abc} \Theta\gamma^c \gamma^7 \Theta ((\vartheta\vartheta)^2 - (vv)^2) \\
 & \left. + \frac{4i}{3R^3} e^b e^d \vartheta\gamma_d \gamma^7 \vartheta \vartheta\gamma^a \gamma^7 \vartheta \varepsilon_{abc} v\gamma^c \varepsilon v \right). \tag{3.38}
 \end{aligned}$$

Note that the maximum order of the fermions in the above expressions is ten.

Using the form of F_4 in (2.9) as well as the expressions (3.32) for \mathcal{A}_1 and (3.37) for $i_\Theta H_3$ the quantity relevant for computing the RR three-form potential \mathcal{A}_3 becomes

$$\begin{aligned}
 i_\Theta F_4 + \mathcal{A}_1 i_\Theta H_3 & = - \left(e^b + i\Theta\gamma^b D\Theta + \frac{1}{R^2} e^b (\vartheta\vartheta - vv)^2 \right) \left(e^a + i\Theta\gamma^a D\Theta + \frac{1}{R^2} e^a (\vartheta\vartheta - vv)^2 \right) \varepsilon_{abc} i\Theta\gamma^c D\Theta \\
 & - 2e^3 \left(e^{a'} - \frac{2}{R} e^a \Theta\gamma^{a'} \gamma_a \Theta \right) D\Theta\gamma_{a'} \Theta - \frac{4}{R} e^3 e^a \Theta\gamma_a D\Theta \Theta\Theta \left(1 + \frac{2}{R^2} (vv)^2 \right) \\
 & - \frac{4}{R} \left(e^{a'} - \frac{2}{R} e^c \Theta\gamma^{a'} \gamma_c \Theta \right) e^a \left(e^b + i\Theta\gamma^b D\Theta + \frac{1}{R^2} e^b (\vartheta\vartheta - vv)^2 \right) \Theta\gamma_{a'} \gamma_{ab} \Theta \left(1 + \frac{2}{R^2} (vv)^2 \right) \\
 & - i \left(e^{b'} - \frac{2}{R} e^b \Theta\gamma^{b'} \gamma_b \Theta \right) \left(e^{a'} - \frac{2}{R} e^a \Theta\gamma^{a'} \gamma_a \Theta \right) D\Theta\gamma_{a'b'} \Theta + kl_p \left(A(y) - \frac{2}{R^2} e^a \Theta\gamma^7 \gamma_a \Theta \right) i_\Theta H_3. \tag{3.39}
 \end{aligned}$$

One can now substitute this together with the expression for $i_\Theta H_3$ (3.37) into eq. (2.11) and compute the explicit form of the RR three-form potential \mathcal{A}_3 in this gauge. Since we have not got a reasonably simple expression for \mathcal{A}_3 we shall not present it here.

4 Applications

We can now use the kappa-gauge fixed form of the $AdS_4 \times CP^3$ superbackground of Subsections 3.1 and 3.2 to simplify the actions for the type IIA superstring and D-branes. Let us note that the gauge fixing conditions (3.8) can also be used to simplify the actions for the $D = 11$ superparticle, M2- and M5-branes in the $AdS_4 \times S^7/Z_k$ superbackground (2.4). We shall consider the example of the $D = 11$ superparticle below.

4.1 $D = 11$ superparticle

Let us consider a massless superparticle in the $AdS_4 \times S^7/Z_k$ supergravity background. Recall that when $k = 1, 2$, the supergravity background preserves the maximum number of 32 supersymmetries, while for $k > 2$ it preserves only 24. The superparticle action in the complete superspace with 32 Θ is constructed using the supervielbeins of the $O\text{Sp}(8|4)/\text{SO}(7) \times \text{SO}(1, 3) \times Z_k$ supercoset derived in [4]

$$\begin{aligned}
 \underline{E}^{\hat{a}} &= E^{\hat{a}}(x, y, \vartheta) + 4iv\gamma^{\hat{a}} \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} Dv + \frac{R}{k} dz E_7^{\hat{a}}(v) \\
 \underline{E}^{a'} &= E^{a'}(x, y, \vartheta) + 2iv \frac{\sinh m}{m} \gamma^{a'} \gamma^5 E(x, y, \vartheta) \\
 \underline{E}^7 &= \frac{R}{k} dz \Phi(v) + R \left(A(x, y, \vartheta) - \frac{4}{R} v \varepsilon \gamma^5 \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} Dv \right) \\
 \underline{E}^{\alpha i} &= \left(\frac{\sinh \mathcal{M}}{\mathcal{M}} (Dv - \frac{2}{k} dz \varepsilon v) \right)^{\alpha i} \\
 \underline{E}^{\alpha a'} &= E^{\alpha a'}(x, y, \vartheta) - \frac{8}{R} E^{\beta a'} \left(\gamma^5 v \frac{\sinh^2 m/2}{m^2} \right)_{\beta i} v^{\alpha i},
 \end{aligned} \tag{4.1}$$

where z is the 7th, $U(1)$ fiber, coordinate of S^7 , Dv has been given in (2.7) and the eight fermionic coordinates $v^{\alpha i}$ correspond to the eight supersymmetries broken by orbifolding with $k > 2$.

The explicit form of the fermionic supervielbeins in (4.1) and of the connections on $O\text{Sp}(8|4)/\text{SO}(7) \times \text{SO}(1, 3) \times Z_k$ are not required for the construction of the Brink-Schwarz superparticle action but one needs them for the construction of the pure-spinor superparticle action in curved superbackgrounds, so we present also the form of the spin-connection below.

The $\text{SO}(1, 3)$ connection is

$$\underline{\Omega}^{\hat{a}\hat{b}} = \Omega^{\hat{a}\hat{b}}(x, y, \vartheta) + \frac{8}{R} v \gamma^{\hat{a}\hat{b}} \gamma^5 \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \left(Dv - \frac{2}{k} dz \varepsilon v \right) \tag{4.2}$$

and the $\text{SO}(7)$ connection is

$$\begin{aligned}
 \underline{\Omega}^{a'b'} &= \Omega^{a'b'}(x, y, \vartheta) - \frac{1}{R} \underline{E}^7 J^{a'b'} - \frac{2}{R} v \frac{\sinh m}{m} \gamma^{a'b'} \gamma^5 E, \\
 \underline{\Omega}^{a'7} &= \frac{1}{R} \left(\underline{E}^{b'} - 4iv \frac{\sinh m}{m} \gamma^{b'} \gamma^5 E \right) J_{b'}^{a'}.
 \end{aligned} \tag{4.3}$$

The functions and forms appearing in (4.1)–(4.3) are defined in appendix B.

The first order form of the action for the massless superparticle in the $\text{OSp}(8|4)/\text{SO}(7) \times \text{SO}(1,3) \times Z_k$ superbackground is

$$S = \int d\tau \left(P_{\underline{A}} \underline{E}_{\tau}^{\underline{A}} + \frac{e}{2} P_{\underline{A}} P_{\underline{B}} \eta^{\underline{AB}} \right), \quad (4.4)$$

where $P_{\underline{A}}$ ($\underline{A} = 0, 1, \dots, 10$) is the particle momentum, $e(\tau)$ is the Lagrange multiplier which ensures the mass shell condition $P^2 = 0$ and

$$\underline{E}_{\tau}^{\underline{A}} = \partial_{\tau} Z^{\underline{M}} \underline{E}_{\underline{M}}^{\underline{A}}, \quad Z^{\underline{M}} = (x, y, z, \vartheta, v)$$

is the pullback to the worldline of the supervielbeins (4.1). The action is invariant under local worldline diffeomorphisms and under the fermionic kappa-symmetry transformations

$$\delta Z^{\underline{M}} \underline{E}_{\underline{M}}^{\alpha} = P^{\underline{A}} (\Gamma_{\underline{A}} \kappa)^{\alpha}, \quad \delta Z^{\underline{M}} \underline{E}_{\underline{M}}^{\underline{A}} = 0, \quad (4.5)$$

$$\delta e = -4i \underline{E}_{\tau}^{\alpha} \kappa_{\alpha}, \quad \delta P_{\underline{A}} = \delta Z^{\underline{M}} \underline{\Omega}_{\underline{M}\underline{A}}^{\underline{B}} P_{\underline{B}}. \quad (4.6)$$

Inserting in the action the expressions for the vielbeins (4.1), we get

$$\begin{aligned} S = \int d\tau & \left[P_{\hat{a}} \left(E_{\tau}^{\hat{a}} + 4iv\gamma^{\hat{a}} \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} D_{\tau} v - 8iv\gamma^{\hat{a}} \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \varepsilon v \frac{\partial_{\tau} z}{k} \right) \right. \\ & + P_{a'} \left(E_{\tau}^{a'} + 2iv \frac{\sinh m}{m} \gamma^{a'} \gamma^5 E_{\tau} \right) \\ & \left. + P_7 \left(R \left(\frac{\partial_{\tau} z}{k} + A \right) - 4v\varepsilon\gamma^5 \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \left(D_{\tau} v - 2\varepsilon v \frac{\partial_{\tau} z}{k} \right) \right) + \frac{e}{2} P_{\underline{A}} P_{\underline{B}} \eta^{\underline{AB}} \right]. \end{aligned} \quad (4.7)$$

The action (4.7) can be simplified by eliminating some or all pure-gauge fermionic modes using the kappa-symmetry transformations (4.5). For instance, when the momentum of the particle is non-zero along a CP^3 direction inside S^7 , the projectors \mathcal{P}_6 and \mathcal{P}_2 , defined in eqs. (A.11) and (A.14), do not commute with the kappa-symmetry projector (4.5) and one can use e.g. the 16 kappa-symmetry transformations to eliminate 16 of the 24 ϑ . After such a gauge fixing the action will contain 8 remaining ϑ and 8 v .

Alternatively, by partially gauge fixing the kappa-symmetry one can eliminate all eight v keeping 24 ϑ . In the latter case the action reduces to the form in which it describes the dynamics of a superparticle in a superspace with 11 bosonic coordinates and 24 fermionic ones. This superspace has been introduced in [4] as a Hopf fibration of the supercoset $\text{OSp}(6|4)/\text{U}(3) \times \text{SO}(1,3)$. It is the supercoset

$$\frac{\text{OSp}(6|4) \times \text{U}(1)}{\text{U}(3) \times \text{SO}(1,3) \times Z_k}. \quad (4.8)$$

The geometry of (4.8) is described by the supervielbeins

$$\begin{aligned} \hat{E}^{\hat{a}} &= E^{\hat{a}}(x, y, \vartheta), \\ \hat{E}^{a'} &= E^{a'}(x, y, \vartheta), \end{aligned} \quad (4.9)$$

$$\begin{aligned}\hat{E}^7 &= R\left(\frac{dz}{k} + A(x, y, \vartheta)\right), \\ \hat{E}^{\alpha\alpha'} &= E^{\alpha\alpha'}(x, y, \vartheta),\end{aligned}$$

where (as already mentioned) the explicit form of the right-hand sides of (4.9) are given in (B.1). Notice that now z appears only in the vielbein \hat{E}^7 along the $U(1)$ -fiber direction of S^7 .

The first order form of the superparticle action in the superspace (4.9) is

$$\begin{aligned}S &= \int d\tau \left(P_{\hat{a}} \hat{E}_{\tau}^{\hat{a}} + P_{a'} \hat{E}_{\tau}^{a'} + P_7 \hat{E}_{\tau}^7 + \frac{e}{2} P_{\underline{A}} P_{\underline{B}} \eta^{AB} \right) \\ &= \int d\tau \left(P_{\hat{a}} E_{\tau}^{\hat{a}} + P_{a'} E_{\tau}^{a'} + P_7 R \left(\frac{\partial_{\tau} z}{k} + A_{\tau} \right) + \frac{e}{2} P_{\underline{A}} P_{\underline{B}} \eta^{AB} \right),\end{aligned}\quad (4.10)$$

where now

$$\hat{E}_{\tau}^{\underline{A}} = \partial_{\tau} Z^{\mathcal{M}} \hat{E}_{\mathcal{M}}^{\underline{A}} + \partial_{\tau} z \hat{E}_z^{\underline{A}}, \quad Z^{\mathcal{M}} = (x, y, \vartheta)$$

is the pullback to the worldline of the supervielbeins (4.9).

It is easy to reduce the action (4.10) to $D = 10$. Once it is done, one obtains the action for a D0-brane moving in the supercoset $OSp(6|4)/U(3) \times SO(1, 3)$.

As we have mentioned above, the action (4.10) describes a superparticle which has a non-zero momentum along the CP^3 base of the S^7 bundle. This is required by the consistency of the kappa-symmetry gauge fixing condition $v = 0$. To describe other possible classical motions of the superparticle, *e.g.* when $P^{a'} = 0$, one should chose a different kappa-symmetry gauge.

For instance, if the superparticle has a non-zero spacial momentum along the 7th, fiber direction, of S^7 , one can use the gauge fixing condition corresponding to that of Subsection 3.1. In this case, in virtue of the gauge-fixed expressions of appendix C, the action (4.7) simplifies to

$$\begin{aligned}S &= \int d\tau \left[P_a \left(e_{\tau}^a(x) + i\vartheta \gamma^a D_{\tau} \vartheta + iv \gamma^a D_{\tau} v - 2iv \gamma^a \varepsilon v \frac{\partial_{\tau} z}{k} \right) \right. \\ &\quad \left. + P_{a'} e_{\tau}^{a'}(y) + P_3 e_{\tau}^3(x) + P_7 R \left(\frac{\partial_{\tau} z}{k} + A_{\tau}(y) \right) + \frac{e}{2} P_{\underline{A}} P_{\underline{B}} \eta^{AB} \right].\end{aligned}\quad (4.11)$$

The dimensional reduction of the $D = 11$ superparticle action (4.11) along z results in the kappa-symmetry gauge-fixed action which describes an arbitrary motion of the type IIA D0-brane in $AdS_4 \times CP^3$ superspace.

Before considering the D0-brane, let us note that the action (4.7) is the most appropriate starting point for the construction of the pure-spinor formulation of the $D = 11$ superparticle in the $AdS_4 \times CP^3$ supergravity background. The pure-spinor condition $\lambda \Gamma^{\underline{A}} \lambda = 0$ in $D = 11$ implies that the 32-component bosonic pure spinor λ^{α} has 23 independent components [37, 38]. This counting ensures the correct number of bosonic and fermionic degrees of freedom.

In the cases of the actions (4.10) and (4.11) that describe a particle motion in the reduced superspaces, one can also develop pure-spinor formulations in which the pure

spinor λ , in addition, is subject to the same constraint as the one imposed on Θ by kappa-symmetry gauge fixing, e.g. $\mathcal{P}_2\lambda = 0$ in the case $v = \mathcal{P}_2\Theta = 0$. This guarantees the correct counting of the degrees of freedom in the pure-spinor formulation (similar to the cases considered in [11, 12]). That is, the difference between the bosonic and fermionic degrees of freedom remains the same. Indeed, in the case of the pure-spinor formulation of the massless $D = 11$ superparticle [37] there are 11 bosonic $X^{\underline{A}}$ plus 23 pure spinor degrees of freedom and 32 fermionic Θ , while in the above example of the reduced pure spinor formulation the pure spinor effectively contains $23 - 8 = 15$ degrees of freedom against 24 fermionic ones, while the number of X remains the same.

When the pure spinor formulations of the superparticle in reduced superspaces correspond to the kappa-gauge fixed versions of the Brink-Schwarz superparticle whose consistency is limited to particular subsectors of the classical configuration space of the full theory, one may expect that the former will also describe only subsectors of the pure-spinor superparticle model formulated in the complete superspace with 32 fermionic coordinates. As in the case of the pure-spinor type IIA superstring in $AdS_4 \times CP^3$ [11, 12], these issues require additional analysis.

4.2 D0-brane

To obtain the action for the $D0$ -brane by dimensional reduction of the $D = 11$ superparticle action, one should first perform the appropriate Lorentz transformation of the $D = 11$ supervielbeins (as was explained in [4]) and make a corresponding redefinition of the particle momentum. We shall not perform this dimensional reduction procedure since the result is well known. The $D0$ -brane action has the following first order form in the type IIA superbackground in the string frame (see eqs. (2.6) and (2.8))

$$S = \int d\tau e^{-\phi} \left(P_A \mathcal{E}_\tau^A + \frac{e}{2} (P_A P_B \eta^{AB} + m^2) \right) + m \int \mathcal{A}_1, \quad (4.12)$$

where m is the mass of the particle and the second term describes its coupling to the RR one-form potential \mathcal{A}_1 .

Integrating out the momenta P_A and the auxiliary field $e(\tau)$ we arrive at the action

$$S = -m \int d\tau e^{-\phi} \sqrt{-\mathcal{E}_\tau^A \mathcal{E}_\tau^B \eta_{AB}} + m \int \mathcal{A}_1. \quad (4.13)$$

The action (4.13) is invariant under worldline diffeomorphisms and the kappa-symmetry transformations (to verify the kappa-symmetry one needs the superspace constraints on the torsion T^A and on F_2 given in appendix A.4)

$$\begin{aligned} \delta_\kappa Z^{\mathcal{M}} \mathcal{E}_{\mathcal{M}}^\alpha &= \frac{1}{2} (1 + \Gamma)^\alpha_{\underline{\beta}} \kappa^{\underline{\beta}}(\tau), & \underline{\alpha} &= 1, \dots, 32, & Z^{\mathcal{M}} &= (x, y, \vartheta, v) \\ \delta_\kappa Z^{\mathcal{M}} \mathcal{E}_{\mathcal{M}}^A &= 0, & A &= 0, 1, \dots, 9 \end{aligned} \quad (4.14)$$

where

$$\Gamma = \frac{1}{\sqrt{-\mathcal{E}_\tau^2}} \mathcal{E}_\tau^A \Gamma_A \Gamma_{11}, \quad \Gamma^2 = 1. \quad (4.15)$$

Comparing the form of the kappa-symmetry projector matrix (4.15) with the kappa-symmetry gauge fixing condition of Subsection 3.1, we see that $\gamma = \Gamma^0 \Gamma^1 \Gamma^2$ (introduced in eq. (3.9)) does not commute with Γ in (4.15) provided that the energy $P^0 \sim \mathcal{E}_\tau^0$ of the massive particle is nonzero, which is always the case. Thus to simplify, e.g. the first order action (4.12) we can use the gauge fixed form of the supervielbeins and the RR one-form of Subsection 3.1. The action takes the following explicit form, with an appropriately rescaled Lagrange multiplier $e(\tau)$,

$$\begin{aligned}
 S = \left(\frac{R}{kl_p}\right)^{-1} \int d\tau \left[\left(P_{a'} e_{\tau'}^{a'}(y) + P_3 e_{\tau}^3(x) \right) \left(1 + \frac{6}{R^2} (vv)^2 \right) \right. \\
 \left. + P_a (e_{\tau}^a(x) + i\Theta \gamma^a D_{\tau} \Theta) \left(1 + \frac{8}{R^2} (vv)^2 \right) + \frac{e(\tau)}{2} (P_A P_B \eta^{AB} + m^2) \right] \\
 + mkl_p \int \left(A(y) - \frac{2i}{R^2} (e^a(x) + i\Theta \gamma^a D\Theta) v \varepsilon \gamma_a v \right). \tag{4.16}
 \end{aligned}$$

This action contains fermionic terms up to the 6th order in $\Theta = (\vartheta, v)$.

4.3 The fundamental string

In this section we use the geometry discussed above to construct the Green-Schwarz model for the fundamental string. We will first review the form of the superstring sigma model without gauge fixing and then impose the gauge fixing of the kappa-symmetry. This will provide a calculable sigma model.

The action for the Green-Schwarz superstring has the following form

$$S = -\frac{1}{4\pi\alpha'} \int d^2\xi \sqrt{-h} h^{IJ} \mathcal{E}_I^A \mathcal{E}_J^B \eta_{AB} - \frac{1}{2\pi\alpha'} \int B_2, \tag{4.17}$$

where ξ^I ($I, J = 0, 1$) are the worldsheet coordinates, $h_{IJ}(\xi)$ is a worldsheet metric and B_2 is the pull-back to the worldsheet of the NS-NS 2-form.

The kappa-symmetry transformations which leave the superstring action (4.17) invariant are

$$\delta_{\kappa} Z^{\mathcal{M}} \mathcal{E}_{\mathcal{M}}^{\alpha} = \frac{1}{2} (1 + \Gamma)_{\underline{\beta}}^{\alpha} \kappa^{\underline{\beta}}(\xi), \tag{4.18}$$

$$\delta_{\kappa} Z^{\mathcal{M}} \mathcal{E}_{\mathcal{M}}^A = 0, \tag{4.19}$$

where $\kappa^{\alpha}(\xi)$ is a 32-component spinor parameter, $\frac{1}{2}(1 + \Gamma)_{\underline{\beta}}^{\alpha}$ is a spinor projection matrix with

$$\Gamma = \frac{1}{2\sqrt{-\det g_{IJ}}} \epsilon^{IJ} \mathcal{E}_I^A \mathcal{E}_J^B \Gamma_{AB} \Gamma_{11}, \quad \Gamma^2 = 1, \tag{4.20}$$

and the auxiliary worldsheet metric h^{IJ} transforms as follows

$$\begin{aligned}
 \delta_{\kappa} (\sqrt{-h} h^{IJ}) & \tag{4.21} \\
 = 2i \sqrt{-h} (h^{IJ} g^{KL} - 2h^{K(I} g^{J)L}) & \left(\delta_{\kappa} Z^{\mathcal{M}} \mathcal{E}_{\mathcal{M}} \Gamma_A \mathcal{E}_K \mathcal{E}_L^A + \frac{1}{2} g_{KL} \delta_{\kappa} Z^{\mathcal{M}} \mathcal{E}_{\mathcal{M}}^{\alpha i} \lambda_{\alpha i} \right)
 \end{aligned}$$

$$\begin{aligned}
 & -2i\sqrt{-h} \frac{h^{IK'} g_{K'L'} h^{L'J} - \frac{1}{2} h^{IJ} h^{K'L'} g_{K'L'}}{\frac{1}{2} h^{K'L'} g_{K'L'} + \sqrt{\frac{g}{h}}} g^{KL} \\
 & \times \left(\delta_\kappa Z^\mathcal{M} \mathcal{E}_\mathcal{M} \Gamma_A \mathcal{E}_K \mathcal{E}_L^A + \frac{1}{2} g_{KL} \delta_\kappa Z^\mathcal{M} \mathcal{E}_\mathcal{M}{}^{\alpha i} \lambda_{\alpha i} \right)
 \end{aligned}$$

where

$$g_{IJ}(\xi) = \mathcal{E}_I^A \mathcal{E}_J^B \eta_{AB}, \quad g^{IJ} \equiv (g_{IJ})^{-1} \quad (4.22)$$

is the induced metric on the worldsheet of the string that on the mass shell coincides with the auxiliary metric $h_{IJ}(\xi)$ modulo a conformal factor. Finally, $g = \det g_{IJ}$ and $h = \det h_{IJ}$.

Using the identity

$$h^{IJ} g_{JK} h^{KL} g_{LI} - \frac{1}{2} (h^{IJ} g_{IJ})^2 \equiv \frac{1}{2} (h^{IJ} g_{IJ})^2 - 2 \frac{g}{h} \quad (4.23)$$

one can check that eq. (4.21) multiplied by g_{IJ} results in

$$\delta_\kappa (\sqrt{-h} h^{IJ}) g_{IJ} = 4i(\sqrt{-g} g^{KL} - \sqrt{-h} h^{KL}) \delta_\kappa Z^\mathcal{M} \mathcal{E}_\mathcal{M}{}^\alpha \left(\Gamma_A \mathcal{E}_K \mathcal{E}_L^A + \frac{1}{2} g_{KL} \lambda \right)_\alpha, \quad (4.24)$$

which together with the variation (4.18) and (4.19) of the superspace coordinates insures the invariance of the action (4.17).

Comparing the form of the kappa-symmetry projector (4.18) with the kappa-symmetry gauge fixing condition of Subsection 3.1 we see that this gauge choice is admissible when the string moves in such a way that the projection of its worldsheet on the $3d$ subspace along the directions e^a ($a = 0, 1, 2$) of the target space is a non-degenerate two-dimensional time-like surface. Thus, it can be used to analyze the string dynamics in the sector which is not reachable by the supercoset model of [9–11]. The latter is obtained from the action (4.17) by gauge fixing to zero the eight fermions v , which is only possible when the string worldsheet extends in the CP^3 directions.

In the gauge of Subsection 3.1 we insert into the action (4.17) the expressions (3.19) and (3.26) for the supervielbeins and B_2 . This results in an action that contains fermionic terms only up to the 8th order in $\Theta = (\vartheta, v)$ ⁴

$$\begin{aligned}
 S = & -\frac{1}{4\pi\alpha'} \frac{R}{kl_p} \int d^2\xi \sqrt{-h} h^{IJ} \left[\left(e_I^{a'} e_J^{b'} \delta_{a'b'} + e_I^3 e_J^3 \right) \left(1 - \frac{6}{R^2} (vv)^2 \right) \right. \\
 & \left. + (e_I^a + i\Theta\gamma^a D_I\Theta) (e_J^b + i\Theta\gamma^b D_J\Theta) \eta_{ab} \left(1 - \frac{2}{R^2} (vv)^2 \right) \right] \\
 & -\frac{1}{2\pi\alpha'} \frac{R}{kl_p} \int \left[e^3 \Theta\gamma^7 D\Theta + i e^{a'} \Theta\gamma_{a'}\gamma^7 D\Theta - \frac{2i}{R} e^3 e^{a'} \vartheta\gamma_{a'}\gamma^7 v - \frac{1}{R} e^{b'} e^{a'} \Theta\gamma_{a'b'}\gamma^7 v \right. \\
 & \left. - \frac{i}{R} (e^b + i\Theta\gamma^b D\Theta) (e^a + i\Theta\gamma^a D\Theta) v\gamma_{ab} \varepsilon v \right].
 \end{aligned} \quad (4.25)$$

⁴The factor in front of the action is unconventional due to our normalization of the vielbeins, which comes from the dimensional reduction of the eleven-dimensional geometry. More conventional, unit radius string frame vielbeins $(\hat{e}^{\hat{a}}, \hat{e}^{a'})$ can be introduced by the following rescaling

$$(e^{\hat{a}}, e^{a'}) = \left(\frac{R}{kl_p} \right)^{-1/2} \left(\frac{R}{2} \hat{e}^{\hat{a}}, R \hat{e}^{a'} \right).$$

Then the factor in front of the action becomes $\frac{R^2}{4\pi\alpha'} = \frac{(R/l_p)^3}{4\pi k}$.

To avoid possible confusion, let us remind the reader that in eqs. (4.25)–(4.29) the covariant derivative $D\Theta \equiv (D_8 v, D_{24}\vartheta)$ is defined in eqs. (3.21). Actually, in (4.25) the vielbein e^3 does not contribute to the covariant derivative and the connection ω^{ab} is zero along the $3d$ Minkowski boundary of AdS_4 , for the vielbeins chosen as in eqs. (3.5). It is not hard to check that the action (4.25) is invariant under twelve ‘linearly realized’ supersymmetry transformations

$$\delta\vartheta = \epsilon, \quad \delta e^a = -i\epsilon\gamma^a D_{24}\vartheta$$

with parameters $\epsilon = \frac{1}{2}(1 + \gamma)\epsilon$ being CP^3 Killing spinors

$$D_{24}\epsilon = \mathcal{P}_6 \left(d - \frac{1}{R}e^3 + \frac{i}{R}e^{a'}\gamma_{a'} - \frac{1}{4}\omega^{ab}\gamma_{ab} - \frac{1}{4}\omega^{a'b'}\gamma_{a'b'} \right) \epsilon = 0.$$

The other twelve supersymmetries of the $OSp(6|4)$ isometries of (4.25) are non-linearly realized on the worldsheet fields and include compensating kappa-symmetry transformations required to maintain the gauge $\vartheta = \frac{1}{2}(1 + \gamma)\vartheta$.

The action (4.25) is slightly more complicated than the action for the $AdS_5 \times S^5$ superstring in the analogous kappa-symmetry gauge [15–17], that contains fermions only up to the fourth order, since $AdS_4 \times CP^3$ is less supersymmetric than $AdS_5 \times S^5$. The action (4.25) takes a form similar to that of [15–17] when we formally put the broken supersymmetry fermions $v^{\alpha i}$ to zero.

As in the case of the string in $AdS_5 \times S^5$ it is possible to simplify the action further by performing a T-duality transformation on the worldsheet [17]. Following [17] we first rewrite the part of the action (4.25) containing the vielbeins e^a in the first order form

$$S_1 = \frac{1}{2\pi\alpha'} \frac{R}{kl_p} \int d^2\xi \quad (4.26)$$

$$\times \left[P_a^I (e_I^a + i\Theta\gamma^a D_I\Theta) + \frac{1 - \frac{6}{R^2}(vv)^2}{2\sqrt{-h}} P_a^I P_b^J h_{IJ} \eta^{ab} - \frac{i}{R} \frac{\varepsilon_{IJ}}{-h} P_a^I P_b^J v\gamma^{ab}\varepsilon v \right].$$

The equations of motion for the momenta P_a^I imply that

$$P_a^I = -\sqrt{-h} \left(1 - \frac{2}{R^2}(vv)^2 \right) \left(h^{IJ} \eta_{ab} + \frac{2i}{R\sqrt{-h}} \varepsilon^{IJ} v\gamma_{ab}\varepsilon v \right) (e_J^b + i\Theta\gamma^b D_J\Theta). \quad (4.27)$$

Using the explicit form of the AdS_4 vielbeins given in eq. (3.5) and varying the first order action (4.26) with respect to x^a we find that P_a^I is proportional to the conserved current associated with translations along x^a

$$\partial_I \left(\frac{r^2}{R^2} P_a^I \right) = 0 \quad \Rightarrow \quad P_a^I = \frac{R^2}{r^2} \varepsilon^{IJ} \partial_J \tilde{x}_a \equiv \varepsilon^{IJ} \tilde{e}_{Ja}. \quad (4.28)$$

If we now substitute eq. (4.28) into (4.26) the T-dualized version of the action (4.25) for the string in $AdS_4 \times CP^3$ takes the form

$$S = -\frac{1}{4\pi\alpha'} \frac{R}{kl_p} \int d^2\xi \sqrt{-h} h^{IJ} \left(\tilde{e}_I^a \tilde{e}_J^b \eta_{ab} + e_I^3 e_J^3 + e_I^{a'} e_J^{b'} \delta_{a'b'} \right) \left(1 - \frac{6}{R^2}(vv)^2 \right) \quad (4.29)$$

$$\begin{aligned}
 & - \frac{1}{2\pi\alpha'} \frac{R}{kl_p} \int \left[e^3 \Theta \gamma^\tau D\Theta + i e^{a'} \Theta \gamma_{a'} \gamma^\tau D\Theta - \frac{2i}{R} e^3 e^{a'} \vartheta \gamma_{a'} \gamma^\tau v - \frac{1}{R} e^{b'} e^{a'} \Theta \gamma_{a'b'} \gamma^\tau v \right. \\
 & \quad \left. + i \tilde{e}^a \Theta \gamma_a D\Theta - \frac{i}{R} \tilde{e}^a \tilde{e}^b v \gamma_{ab} \varepsilon v \right].
 \end{aligned}$$

Note that in the T-dualized action the fermionic kinetic terms appear only in the Wess-Zumino term and that there are now terms of at most fourth order in fermions. Note also that the first (induced metric) term of (4.29) acquires a common factor $(1 - \frac{6}{R^2}(v v)^2)$ in contrast to the corresponding terms in the original action (4.25).

To preserve the conformal invariance of the dual action at the quantum level one should add to it a dilaton term $\int R^{(2)} \tilde{\phi}$ (where $R^{(2)}$ is the worldsheet curvature), which is induced by the functional integration of P_a^I when passing to the dual action (see [17, 39, 40] for details). Here we should point out that in our case the original $AdS_4 \times CP^3$ superbackground already has a non-trivial dilaton which depends on v (see eqs. (2.12) and (3.15)).

The following comment is now in order. As in the $AdS_5 \times S^5$ case [17, 22, 24], upon the T-duality along the three translational directions x^a of AdS_4 the purely bosonic (classically integrable) $AdS_4 \times CP^3$ sector of the type IIA superstring sigma model maps into an equivalent sigma model on a dual AdS_4 space, both models sharing the same integrable structure [22]. The situation with the fermionic sector of the $AdS_4 \times CP^3$ superstring is, however, different due to the fact that there is less supersymmetry than in the $AdS_5 \times S^5$ case.

In the case of the $AdS_5 \times S^5$ superstring sigma model, one can accompany the above bosonic T-duality transformation by a fermionic one along fermionic directions in (complexified) superspace which have translational isometries [23, 24]. This compensates the dilaton term generated by the bosonic T-duality and maps the $AdS_5 \times S^5$ superstring action to an equivalent (dual) one, which is also integrable.⁵ However, in the $AdS_4 \times CP^3$ case under consideration the fermionic directions in superspace parametrized by v do not have translational isometries, since the action (4.25) or (4.29) has v -dependent fermionic terms which do not contain worldsheet derivatives. This just reflects the fact that the fermionic modes v correspond to the broken supersymmetries of the superbackground.

As far as the T-dualization of the supersymmetric fermionic modes ϑ is concerned, it might be, in principle, possible (at least in the absence of v) if, as in the case of the $SU(2, 2|4)$ superstring sigma model [23, 24], there existed a realization of the $OSp(6|4)$ superalgebra in which 12 of the 24 (complex conjugate) supersymmetry generators squared to zero and formed a representation of the bosonic subalgebra of $OSp(6|4)$. In other words the possibility of T-dualizing part of fermionic modes ϑ (in the absence of v) is related to the question of the existence of a chiral superspace representation of the superalgebra $OSp(6|4)$. Such a realization of $OSp(6|4)$ seems not to exist. In fact, it has been argued in [25] that the $OSp(6|4)$ supercoset subsector of the Green-Schwarz superstring in $AdS_4 \times CP^3$ does not have any fermionic T-duality symmetry since in $OSp(6|4)$ the dimension of the representation of the supercharges under the R-symmetry is odd. The absence of the

⁵In [22–24] it has been shown that this duality property of the $AdS_5 \times S^5$ superstring is related to earlier observed dual conformal symmetry of maximally helicity violating amplitudes of the $\mathcal{N} = 4$ super-Yang-Mills theory and to the relation between gluon scattering amplitudes and Wilson loops at strong and weak coupling.

fermionic T-duality of the superstring in $AdS_4 \times CP^3$ may have interesting manifestations in particular features of the AdS_4/CFT_3 holography.

The gauge-fixed actions (4.25) or (4.29) can be used for studying different aspects of the AdS_4/CFT_3 correspondence and integrability on both of its sides, in particular, for making two- and higher-loop string computations for testing the Bethe ansatz and the S-matrix [41]–[49] in the dual planar $\mathcal{N} = 6$ superconformal Chern-Simons-matter theory, which would extend the analysis of [50]–[21, 66] and others.

4.4 D2-branes

Let us now consider the effective worldvolume theory of probe D2-branes moving in the $AdS_4 \times CP^3$ superbackground. This can be derived from the action for D -branes in a generic type IIA superbackground [67–69] by substituting the explicit form of the $AdS_4 \times CP^3$ supergeometry (2.6)–(2.11).

The action for a D2-brane in a generic type IIA supergravity background in the string frame has the following form

$$S = -T \int d^3\xi e^{-\phi} \sqrt{-\det(g_{IJ} + \mathcal{F}_{IJ})} + T \int (\mathcal{A}_3 + \mathcal{A}_1 \mathcal{F}_2), \quad (4.30)$$

where T is the tension of the D2-brane, $\phi(Z)$ is the dilaton superfield,

$$g_{IJ}(\xi) = \mathcal{E}_I^A \mathcal{E}_J^B \eta_{AB} \quad I, J = 0, 1, 2; \quad A, B = 0, 1, \dots, 9 \quad (4.31)$$

is the induced metric on the D2-brane worldvolume with $\mathcal{E}_I^A = \partial_I Z^M \mathcal{E}_{\mathcal{M}}^A$ being the pullbacks of the vector supervielbeins of the type IIA $D = 10$ superspace and

$$\mathcal{F}_2 = d\mathcal{V} - B_2 \quad (4.32)$$

is the field strength of the worldvolume Born-Infeld gauge field $\mathcal{V}_I(\xi)$ extended by the pullback of the NS-NS two-form. \mathcal{A}_1 and \mathcal{A}_3 are the pullbacks of the type IIA supergravity RR superforms (2.8) and (2.11).

Provided that the superbackground satisfies the IIA supergravity constraints, the action (4.30) is invariant under kappa-symmetry transformations of the superstring coordinates $Z^M(\xi)$ of the form (4.18), (4.19), together with

$$\delta_\kappa \mathcal{F}_{IJ} = -\mathcal{E}_J^B \mathcal{E}_I^A \delta_\kappa Z^M \mathcal{E}_{\mathcal{M}}^\alpha H_{\alpha AB} = -4i \mathcal{E}_{[I}^A \mathcal{E}_{J]} \Gamma_A \Gamma_{11} \delta_\kappa \mathcal{E} - 2i \mathcal{E}_J^B \mathcal{E}_I^A \delta_\kappa \mathcal{E} \Gamma_{AB} \Gamma_{11} \lambda, \quad (4.33)$$

where $\delta_\kappa \mathcal{E} = \delta_\kappa Z^M \mathcal{E}_{\mathcal{M}}^\alpha$.

In the case of the $D2$ -brane the matrix Γ has the form

$$\begin{aligned} \Gamma &= -\frac{1}{\sqrt{-\det(g + \mathcal{F})}} \varepsilon^{IJK} \left(\frac{1}{3!} \mathcal{E}_I^A \mathcal{E}_J^B \mathcal{E}_K^C \Gamma_{ABC} + \frac{1}{2} \mathcal{F}_{IJ} \mathcal{E}_K^A \Gamma_A \Gamma_{11} \right) \\ &= -\frac{1}{3! \sqrt{-\det(g + \mathcal{F})}} \varepsilon^{IJK} \mathcal{E}_I^A \mathcal{E}_J^B \mathcal{E}_K^C \Gamma_{ABC} \left(1 + \frac{1}{2} \mathcal{F}^{IJ} \mathcal{E}_I^A \mathcal{E}_J^B \Gamma_{AB} \Gamma_{11} \right). \end{aligned} \quad (4.34)$$

4.4.1 D2 filling $AdS_2 \times S^1$ inside of AdS_4

Let us consider the D2-brane configuration which corresponds to a disorder loop operator in the ABJM theory [29]. The 1/2 BPS static solution of the equations of motion of the D2-brane on $AdS_2 \times S^1$ in the metric

$$ds^2 = \frac{R_{CP^3}^2}{4u^2}(-dx^0 dx^0 + dr^2 + r^2 d\varphi^2 + du^2) + R_{CP^3}^2 ds_{CP^3}^2, \quad (4.35)$$

where

$$ds_{CP^3}^2 = \frac{1}{4} \left[d\alpha^2 + \cos^2 \frac{\alpha}{2} (d\vartheta_1^2 + \sin^2 \vartheta_1 d\varphi_1^2) + \sin^2 \frac{\alpha}{2} (d\vartheta_2^2 + \sin^2 \vartheta_2 d\varphi_2^2) + \sin^2 \frac{\alpha}{2} \cos^2 \frac{\alpha}{2} (d\chi + \cos \vartheta_1 d\varphi_1 - \cos \vartheta_2 d\varphi_2)^2 \right],$$

is characterized by the following embedding of the brane worldvolume

$$\xi^0 = x^0, \quad \xi_1 = u, \quad \xi_2 = \varphi, \quad r = a \xi_1 \quad (4.36)$$

which is supported by the non-zero (electric) Born-Infeld field strength

$$F = E \frac{dx^0 \wedge du}{u^2}, \quad E = \frac{R_{CP^3}^2}{4} \sqrt{1 + a^2}, \quad (4.37)$$

where a is an arbitrary constant. Note that the presence of the non-zero DBI flux on the AdS_2 subspace of the D2-brane worldvolume is required to ensure the no-force condition, i.e. vanishing of the classical action (4.30) of this static D2-brane configuration, provided that also an additional BI flux boundary counterterm is added to the action (see [29] for more details). A natural explanation of this boundary term is that it appears in the process of the dualization of the compactified 11th coordinate scalar field of the M2-brane into the BI vector field of the D2-brane.

Note that in [29] this brane configuration was considered in a different coordinate system, in which AdS_4 is foliated with $AdS_2 \times S^1$ slices instead of the flat $R^{1,2}$ slices. This makes manifest the symmetries of the D2-brane configuration. An explicit form of the AdS_4 metric in this slicing is

$$ds_{AdS_4}^2 = \frac{R_{CP^3}^2}{4} (\cosh^2 \psi ds_{AdS_2}^2 + d\psi^2 + \sinh^2 \psi d\varphi^2) \quad (4.38)$$

which is essentially a double analytic continuation of the usual global AdS_4 metric. The static D2-brane configuration is then characterized by the identification of the worldvolume coordinates ξ^a with those of AdS_2 and the S^1 angle φ . However, for our choice of the kappa-symmetry gauge fixing condition the use of the metric in the form (4.35) is more convenient, since the associated AdS_4 vielbeins

$$e^0 = \frac{R_{CP^3}}{2u} dx^0, \quad e^1 = \frac{R_{CP^3}}{2u} dr, \quad e^2 = \frac{R_{CP^3} r}{2u} d\varphi, \quad e^3 = -\frac{R_{CP^3}}{2u} du \quad (4.39)$$

and the spin connection directly satisfy the relations (3.6) and (3.7).

One can be interested in D2-brane bosonic and fermionic fluctuations around this 1/2 BPS static D2-brane solution described by the action (4.30). To simplify the form of the fermionic terms, the kappa-symmetry gauge fixing for the D2-brane wrapping $AdS_2 \times S^1$ can be made in the simplest possible way considered in Subsection 3.1. To get the gauge fixed D2-brane action in this case one should substitute into (4.30) the expressions for the vector supervielbeins (3.19), the RR one-form (3.20) and the three-form (3.29), and the NS-NS two-form (3.26).

4.4.2 D2 at the Minkowski boundary of AdS_4

Let us now consider the supersymmetric effective worldvolume action describing a D2-brane placed at the Minkowski boundary of the AdS_4 space. In this case it is convenient to choose the $AdS_4 \times CP^3$ metric in the form (3.1) or (3.4).

When the D2-brane is at the Minkowski boundary, we take the static gauge $\xi^m = x^m$. The 1/2 BPS ground state of the D2-brane is when its transverse scalar modes are constant and the Born-Infeld field and the fermionic modes are zero. As a consistency check, let us note that with the choice of the background value of the RR 3-form (2.11) and (3.3) and of the corresponding (positive) $D2$ -brane charge (characterized by the plus sign in front of the Wess-Zumino term (4.30)), the action of the ground state of the D2-brane at the Minkowski boundary vanishes. This means that such a brane configuration is stable and does not experience any external force, i.e. it is a BPS state.

If, on the other hand, with the same choice of \mathcal{A}_3 (2.11) and (3.3), we considered an anti- $D2$ -brane carrying a negative \mathcal{A}_3 charge (which would be characterized by a minus sign in front of the Wess-Zumino term in (4.30)), the ground state of this anti- $D2$ -brane at the Minkowski boundary would have a non-zero action

$$S_{\overline{D2}} = -2T e^{-\phi_0} \int d^3x \left(\frac{r}{R_{CP^3}} \right)^6$$

implying that such a solution is unstable (as is well known to be the case for a probe anti-D-brane in a background of D-branes). It is, therefore, important for the consistency of the solution to take care that the relative signs of the RR potential \mathcal{A}_3 and the $D2$ -brane charge (and, as a consequence, the sign of the kappa-symmetry projector) ensure the no-force condition, i.e. vanishing of the static $D2$ -brane action. In the case of M2, M5 and D3-branes at the Minkowski boundary of AdS this issue was discussed in detail in [19].

For the static D2-brane configuration the kappa-symmetry projector (4.34) reduces to

$$P = \frac{1}{2}(1 + \gamma), \quad \gamma = \gamma^0 \gamma^1 \gamma^2 = -\gamma_0 \gamma_1 \gamma_2 \tag{4.40}$$

So the natural choice of the kappa-symmetry gauge fixing condition is

$$\Theta = \frac{1}{2}(1 - \gamma)\Theta, \tag{4.41}$$

i.e. the gauge choice considered in detail in Subsection 3.2. Note that in the case of the D2-brane at the Minkowski boundary we cannot use the simpler condition $\Theta = \frac{1}{2}(1 + \gamma)\Theta$ of Subsection 3.1, because the kappa-symmetry projector (4.40) has the same sign.

Plugging the kappa-symmetry gauge-fixed quantities of Subsection 3.2 into the action (4.30), one can study the properties of the $\text{OSp}(6|4)$ invariant effective $3d$ gauge-matter field theory on the worldvolume of the $D2$ -brane placed at the Minkowski boundary of AdS_4 , which from the point of view of M-theory corresponds to an $M2$ -brane pulled out to a finite distance from a stack of $M2$ -branes probing R^8/Z_k .

The effective theory on the worldvolume of this $D2$ -brane, which describes its fluctuations in $AdS_4 \times CP^3$, is an interacting $d = 3$ gauge Born-Infeld-matter theory possessing the (spontaneously broken) superconformal symmetry $\text{OSp}(6|4)$. The model is superconformally invariant in spite of the presence on the $d = 3$ worldvolume of the dynamical Abelian vector field, since the latter is coupled to the $3d$ dilaton field associated with the radial direction of AdS_4 . The superconformal invariance is spontaneously broken by a non-zero expectation value of the dilaton. An $\mathcal{N} = 3$ superfield model with similar symmetry properties was considered in the appendix of [70]. To establish the explicit relation between the two models one should extract from the superfield action of [70] the component terms describing its physical sector and compare the result with corresponding terms in the $D2$ -brane action.

5 Conclusion

In this paper we have considered the gauge-fixing of kappa-symmetry of the superparticle, superstring and $D2$ -brane actions in the complete $AdS_4 \times CP^3$ superspace which is suitable, in particular, for studying regions of these theories that are not reachable by partially kappa-symmetry gauge fixed models based on the supercoset $\text{OSp}(6|4)/U(3) \times \text{SO}(1, 3)$. The simplified form of these actions can be used to approach various problems of the AdS_4/CFT_3 correspondence. The gauge fixed form of the $AdS_4 \times CP^3$ supergeometry can also be used to consider the actions for higher dimensional $D4$ -, $D6$ - and $D8$ -branes.

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A Main notation and conventions

The convention for the ten and eleven dimensional metrics is the ‘almost plus’ signature $(-, +, \dots, +)$. Generically, the tangent space vector indices are labeled by letters from the

beginning of the Latin alphabet, while letters from the middle of the Latin alphabet stand for curved (world) indices. The spinor indices are labeled by Greek letters.

A.1 AdS_4 space

AdS_4 is parametrized by the coordinates $x^{\hat{m}}$ and its vielbeins are $e^{\hat{a}} = dx^{\hat{m}} e_{\hat{m}}^{\hat{a}}(x)$, $\hat{m} = 0, 1, 2, 3$; $\hat{a} = 0, 1, 2, 3$. The $D = 4$ gamma-matrices satisfy:

$$\{\gamma^{\hat{a}}, \gamma^{\hat{b}}\} = 2\eta^{\hat{a}\hat{b}}, \quad \eta^{\hat{a}\hat{b}} = \text{diag}(-, +, +, +), \quad (\text{A.1})$$

$$\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3, \quad \gamma^5 \gamma^5 = 1. \quad (\text{A.2})$$

The charge conjugation matrix C is antisymmetric, the matrices $(\gamma^{\hat{a}})_{\alpha\beta} \equiv (C\gamma^{\hat{a}})_{\alpha\beta}$ and $(\gamma^{\hat{a}\hat{b}})_{\alpha\beta} \equiv (C\gamma^{\hat{a}\hat{b}})_{\alpha\beta}$ are symmetric and $\gamma_{\alpha\beta}^5 \equiv (C\gamma^5)_{\alpha\beta}$ is antisymmetric, with $\alpha, \beta = 1, 2, 3, 4$ being the indices of a 4-dimensional spinor representation of $SO(1, 3)$ or $SO(2, 3)$.

A.2 CP^3 space

CP^3 is parametrized by the coordinates $y^{m'}$ and its vielbeins are $e^{a'} = dy^{m'} e_{m'}^{a'}(y)$, $m' = 1, \dots, 6$; $a' = 1, \dots, 6$. The $D = 6$ gamma-matrices satisfy:

$$\{\gamma^{a'}, \gamma^{b'}\} = 2\delta^{a'b'}, \quad \delta^{a'b'} = \text{diag}(+, +, +, +, +, +), \quad (\text{A.3})$$

$$\gamma^7 = \frac{i}{6!} \epsilon_{a'_1 a'_2 a'_3 a'_4 a'_5 a'_6} \gamma^{a'_1} \dots \gamma^{a'_6} \quad \gamma^7 \gamma^7 = 1. \quad (\text{A.4})$$

The charge conjugation matrix C' is symmetric and the matrices $(\gamma^{a'})_{\alpha'\beta'} \equiv (C'\gamma^{a'})_{\alpha'\beta'}$ and $(\gamma^{a'b'})_{\alpha'\beta'} \equiv (C'\gamma^{a'b'})_{\alpha'\beta'}$ are antisymmetric, with $\alpha', \beta' = 1, \dots, 8$ being the indices of an 8-dimensional spinor representation of $SO(6)$.

A.3 Type IIA $AdS_4 \times CP^3$ superspace

The type IIA superspace whose bosonic body is $AdS_4 \times CP^3$ is parametrized by 10 bosonic coordinates $X^M = (x^{\hat{m}}, y^{m'})$ and 32-fermionic coordinates $\Theta^\mu = (\Theta^{\mu\mu'})$ ($\mu = 1, 2, 3, 4$; $\mu' = 1, \dots, 8$). These combine into the superspace supercoordinates $Z^{\mathcal{M}} = (x^{\hat{m}}, y^{m'}, \Theta^{\mu\mu'})$. The type IIA supervielbeins are

$$\mathcal{E}^{\mathcal{A}} = dZ^{\mathcal{M}} \mathcal{E}_{\mathcal{M}}^{\mathcal{A}}(Z) = (\mathcal{E}^A, \mathcal{E}^\alpha), \quad \mathcal{E}^A(Z) = (\mathcal{E}^{\hat{a}}, \mathcal{E}^{a'}), \quad \mathcal{E}^\alpha(Z) = \mathcal{E}^{\alpha\alpha'}. \quad (\text{A.5})$$

A.4 Superspace constraints

In our conventions the superspace constraint on the bosonic part of the torsion is

$$T^A = -i\mathcal{E}\Gamma^A\mathcal{E} + i\mathcal{E}^A\mathcal{E}\lambda + \frac{1}{3}\mathcal{E}^A\mathcal{E}^B\nabla_B\phi, \quad (\text{A.6})$$

while the constraints on the RR and NS-NS field strengths are

$$F_2 = -ie^{-\phi}\mathcal{E}\Gamma_{11}\mathcal{E} + 2ie^{-\phi}\mathcal{E}^A\mathcal{E}\Gamma_A\Gamma_{11}\lambda + \frac{1}{2}\mathcal{E}^B\mathcal{E}^A F_{AB}, \quad (\text{A.7})$$

$$F_4 = -\frac{i}{2}e^{-\phi}\mathcal{E}^B\mathcal{E}^A\mathcal{E}\Gamma_{AB}\mathcal{E} + \frac{1}{4!}\mathcal{E}^D\mathcal{E}^C\mathcal{E}^B\mathcal{E}^A F_{ABCD}, \quad (\text{A.8})$$

$$H_3 = -i\mathcal{E}^A\mathcal{E}\Gamma_A\Gamma_{11}\mathcal{E} + i\mathcal{E}^B\mathcal{E}^A\mathcal{E}\Gamma_{AB}\Gamma^{11}\lambda + \frac{1}{3!}\mathcal{E}^C\mathcal{E}^B\mathcal{E}^A H_{ABC}. \quad (\text{A.9})$$

These differ from the conventional string frame constraints by the λ -term in T^A and related terms in F_2 , F_4 and H_3 . This is a consequence of the dimensional reduction from eleven dimensions. They can be brought to a more conventional form by shifting the fermionic supervielbein \mathcal{E}^α by $-\frac{1}{2}\mathcal{E}^A(\Gamma_A\lambda)^\alpha$ accompanied by a related shift in the connection.

The $D = 10$ gamma-matrices Γ^A are given by

$$\begin{aligned} \{\Gamma^A, \Gamma^B\} &= 2\eta^{AB}, & \Gamma^A &= (\Gamma^{\hat{a}}, \Gamma^{a'}), \\ \Gamma^{\hat{a}} &= \gamma^{\hat{a}} \otimes \mathbf{1}, & \Gamma^{a'} &= \gamma^5 \otimes \gamma^{a'}, & \Gamma^{11} &= \gamma^5 \otimes \gamma^7, & a &= 0, 1, 2, 3; & a' &= 1, \dots, 6. \end{aligned} \tag{A.10}$$

The charge conjugation matrix is $\mathcal{C} = C \otimes C'$.

The fermionic variables Θ^α of IIA supergravity carrying 32-component spinor indices of $\text{Spin}(1, 9)$, in the $AdS_4 \times CP^3$ background and for the above choice of the $D = 10$ gamma-matrices, naturally split into 4-dimensional $\text{Spin}(1, 3)$ indices and 8-dimensional spinor indices of $\text{Spin}(6)$, i.e. $\Theta^\alpha = \Theta^{\alpha\alpha'}$ ($\alpha = 1, 2, 3, 4$; $\alpha' = 1, \dots, 8$).

A.5 24 + 8 splitting of 32 Θ

24 of $\Theta^\alpha = \Theta^{\alpha\alpha'}$ correspond to the unbroken supersymmetries of the $AdS_4 \times CP^3$ background. They are singled out by a projector introduced in [6] which is constructed using the CP^3 Kähler form $J_{a'b'}$ and seven 8×8 antisymmetric gamma-matrices (A.3). The 8×8 projector matrix has the following form

$$\mathcal{P}_6 = \frac{1}{8}(6 - J), \tag{A.11}$$

where the 8×8 matrix

$$J = -iJ_{a'b'}\gamma^{a'b'}\gamma^7 \quad \text{such that} \quad J^2 = 4J + 12 \tag{A.12}$$

has six eigenvalues -2 and two eigenvalues 6 , i.e. its diagonalization results in

$$J = \text{diag}(-2, -2, -2, -2, -2, -2, 6, 6). \tag{A.13}$$

Therefore, the projector (A.11) when acting on an 8-dimensional spinor annihilates 2 and leaves 6 of its components, while the complementary projector

$$\mathcal{P}_2 = \frac{1}{8}(2 + J), \quad \mathcal{P}_2 + \mathcal{P}_6 = \mathbf{1} \tag{A.14}$$

annihilates 6 and leaves 2 spinor components.

Thus the spinor

$$\vartheta^{\alpha\alpha'} = (\mathcal{P}_6 \Theta)^{\alpha\alpha'} \iff \vartheta^{\alpha a'} \quad a' = 1, \dots, 6 \tag{A.15}$$

has 24 non-zero components and the spinor

$$\upsilon^{\alpha\alpha'} = (\mathcal{P}_2 \Theta)^{\alpha\alpha'} \iff \upsilon^{\alpha i} \quad i = 1, 2 \tag{A.16}$$

has 8 non-zero components. The latter corresponds to the eight supersymmetries broken by the $AdS_4 \times CP^3$ background.

To avoid confusion, let us note that the index a' on spinors is different from the same index on bosonic quantities. They are related by the usual relation between vector and spinor representations, i.e. given two Spin(6) spinors $\psi_1^{a'}$ and $\psi_2^{a'}$, projected as in (A.15), their bilinear combination $v^{a'} = \psi_1 \mathcal{P}_6 \gamma^{a'} \mathcal{P}_6 \psi_2 = \psi_1^{b'} (\mathcal{P}_6 \gamma^{a'} \mathcal{P}_6)_{b'c'} \psi_2^{c'}$ transforms as a 6-dimensional 'vector'.

B OSp(6|4)/U(3) \times SO(1, 3) supercoset realization and other ingredients of the (10|32)-dimensional $AdS_4 \times CP^3$ superspace

The supervielbeins and the superconnections of the OSp(6|4)/U(3) \times SO(1, 3) supercoset which appear in the definition of the geometric and gauge quantities of the $AdS_4 \times CP^3$ superspace in section 2 are

$$\begin{aligned}
 E^{\hat{a}} &= e^{\hat{a}}(x) + 4i\vartheta\gamma^{\hat{a}} \frac{\sinh^2 \mathcal{M}_{24}/2}{\mathcal{M}_{24}^2} D_{24}\vartheta, \\
 E^{a'} &= e^{a'}(y) + 4i\vartheta\gamma^{a'}\gamma^5 \frac{\sinh^2 \mathcal{M}_{24}/2}{\mathcal{M}_{24}^2} D_{24}\vartheta, \\
 E^{\alpha a'} &= \left(\frac{\sinh \mathcal{M}_{24}}{\mathcal{M}_{24}} D_{24}\vartheta \right)^{\alpha a'}, \\
 \Omega^{\hat{a}\hat{b}} &= \omega^{\hat{a}\hat{b}}(x) + \frac{8}{R}\vartheta\gamma^{\hat{a}\hat{b}}\gamma^5 \frac{\sinh^2 \mathcal{M}_{24}/2}{\mathcal{M}_{24}^2} D_{24}\vartheta, \\
 \Omega^{a'b'} &= \omega^{a'b'}(y) - \frac{4}{R}\vartheta(\gamma^{a'b'} - iJ^{a'b'}\gamma^7)\gamma^5 \frac{\sinh^2 \mathcal{M}_{24}/2}{\mathcal{M}_{24}^2} D_{24}\vartheta, \\
 A &= \frac{1}{8}J_{a'b'}\Omega^{a'b'} = A(y) + \frac{4i}{R}\vartheta\gamma^7\gamma^5 \frac{\sinh^2 \mathcal{M}_{24}/2}{\mathcal{M}_{24}^2} D_{24}\vartheta,
 \end{aligned} \tag{B.1}$$

where

$$\begin{aligned}
 R(\mathcal{M}_{24}^2)^{\alpha a'}{}_{\beta b'} &= 4\vartheta_{b'}^\alpha (\vartheta^{a'}\gamma^5)_\beta - 4\delta_{b'}^{a'}\vartheta^{\alpha c'} (\vartheta\gamma^5)_{\beta c'} \\
 &\quad - 2(\gamma^5\gamma^{\hat{a}}\vartheta)^{\alpha a'} (\vartheta\gamma_{\hat{a}})_{\beta b'} - (\gamma^{\hat{a}\hat{b}}\vartheta)^{\alpha a'} (\vartheta\gamma_{\hat{a}\hat{b}}\gamma^5)_{\beta b'}.
 \end{aligned} \tag{B.2}$$

The derivative appearing in the above equations is defined as

$$D_{24}\vartheta = \mathcal{P}_6 \left(d + \frac{i}{R}e^{\hat{a}}\gamma^5\gamma_{\hat{a}} + \frac{i}{R}e^{a'}\gamma_{a'} - \frac{1}{4}\omega^{\hat{a}\hat{b}}\gamma_{\hat{a}\hat{b}} - \frac{1}{4}\omega^{a'b'}\gamma_{a'b'} \right) \vartheta, \tag{B.3}$$

where $e^{\hat{a}}(x)$, $e^{a'}(y)$, $\omega^{\hat{a}\hat{b}}(x)$, $\omega^{a'b'}(y)$ and $A(y)$ are the vielbeins and connections of the bosonic solution. The U(3)-connection $\Omega^{a'b'}$ satisfies the condition

$$(P^-)_{a'b'}{}^{c'd'}\Omega_{c'd'} = \frac{1}{2}(\delta_{[a'c'}\delta_{b']}{}^{d'} - J_{[a'c'}J_{b']}{}^{d'})\Omega_{c'd'} = 0, \tag{B.4}$$

where $J_{a'b'}$ is the Kähler form on CP^3 .

B.1 Other quantities appearing in the definition of the $AdS_4 \times CP^3$ superspace of section 2

$$R(\mathcal{M}^2)^{\alpha i}{}_{\beta j} = 4(\varepsilon v)^{\alpha i}{}_{\beta j} - 2(\gamma^5 \gamma^{\hat{a}} v)^{\alpha i}{}_{\beta j} - (\gamma^{\hat{a} \hat{b}} v)^{\alpha i}{}_{\beta j}, \quad (\text{B.5})$$

$$(m^2)^{ij} = -\frac{4}{R} v^i \gamma^5 v^j, \quad (\text{B.6})$$

$$\Lambda_{\hat{a}}{}^{\hat{b}} = \delta_{\hat{a}}{}^{\hat{b}} - \frac{R^2}{k^2 l_p^2} \cdot \frac{e^{-\frac{2}{3}\phi}}{e^{\frac{2}{3}\phi} + \frac{R}{kl_p} \Phi} E_{7\hat{a}} E_{7\hat{b}}, \quad (\text{B.7})$$

$$S_{\underline{\beta}}{}^{\underline{\alpha}} = \frac{e^{-\frac{1}{3}\phi}}{\sqrt{2}} \left(\sqrt{e^{\frac{2}{3}\phi} + \frac{R}{kl_p} \Phi} - \frac{R}{kl_p} \frac{E_{7\hat{a}} \Gamma_{\hat{a}} \Gamma_{11}}{\sqrt{e^{\frac{2}{3}\phi} + \frac{R}{kl_p} \Phi}} \right)_{\underline{\beta}}{}^{\underline{\alpha}}$$

$$E_{7\hat{a}}(v) = -\frac{8i}{R} v \gamma^{\hat{a}} \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \varepsilon v, \quad (\text{B.8})$$

$$\Phi(v) = 1 + \frac{8}{R} v \varepsilon \gamma^5 \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \varepsilon v.$$

Let us emphasise that the $SO(2)$ indices $i, j = 1, 2$ are raised and lowered with the unit matrices δ^{ij} and δ_{ij} so that there is actually no difference between the upper and the lower $SO(2)$ indices, $\varepsilon_{ij} = -\varepsilon_{ji}$, $\varepsilon^{ij} = -\varepsilon^{ji}$ and $\varepsilon^{12} = \varepsilon_{12} = 1$.

C Identities for the kappa-projected fermions

When the fermionic variables $\Theta^{\underline{\alpha}} = (\vartheta^{\alpha\alpha'}, v^{\alpha i})$ are subject to the constraint (3.8), the following identities hold.

C.1 Identities involving $v^{\alpha i}$

$$v^i \gamma^5 v^j = v^i \gamma^3 v^j = 0, \quad v^{\alpha i} v^{\beta j} \delta_{ij} = -\frac{1}{4} ((1 \pm \gamma) C^{-1})^{\alpha\beta} v v, \quad (\text{C.1})$$

where $\gamma = \gamma^{012}$ and $v v = \delta_{ij} v^{\alpha i} C_{\alpha\beta} v^{\beta j}$.

Another useful relation is ($\varepsilon^{012} = -\varepsilon_{012} = 1$)

$$v \gamma_{ab} dv = \pm \varepsilon_{abc} v \gamma^c dv, \quad (\text{C.2})$$

which also holds for the kappa-projected ϑ and $d\vartheta$.

Using eqs. (C.1) and (C.2) we find that

$$v \varepsilon \gamma^a v v \varepsilon \gamma_b v = \delta_b^a (v v)^2, \quad v \varepsilon \gamma^{ac} v v \varepsilon \gamma_{cb} v = 2 \delta_b^a (v v)^2, \quad (\text{C.3})$$

$$(m^2)^{ij} = -\frac{4}{R} v^i \gamma^5 v^j = 0 \quad (\text{C.4})$$

and

$$(\mathcal{M}^2 \varepsilon v)^{\alpha i} = 0. \quad (\text{C.5})$$

A similar computation shows that

$$v \varepsilon \gamma^5 \mathcal{M}^2 = 0. \quad (\text{C.6})$$

It is also true in general (i.e. without fixing κ -symmetry) that

$$\mathcal{M}^2 v = 0, \quad v \gamma^5 \mathcal{M}^2 = 0. \quad (\text{C.7})$$

Using the above identities we find that for v satisfying (3.8)

$$\mathcal{M}^2 Dv = \frac{6i}{R^2} \left(E^a \pm \frac{R}{2} \Omega^{a3} \right) (\gamma_a v) v v \quad (\text{C.8})$$

which results in

$$\begin{aligned} 4v \gamma^a \frac{\sinh^2(\mathcal{M}/2)}{\mathcal{M}^2} Dv &= v \gamma^a \left(1 + \frac{1}{12} \mathcal{M}^2 \right) Dv \\ &= v \gamma^a \left(d - \frac{1}{4} \Omega^{bc} \gamma_{bc} \right) v + \frac{i}{2R^2} \left(E^a \pm \frac{R}{2} \Omega^{a3} \right) (vv)^2, \end{aligned} \quad (\text{C.9})$$

where E^a , Ω^{bc} and Ω^{a3} are AdS_4 components of the supervielbein and connection of the supercoset $\text{OSp}(6|4)/\text{U}(3) \times \text{SO}(1,3)$ defined in eqs. (B.1) and the matrix \mathcal{M}^2 is defined in eq. (B.5).

We also find that

$$4v \varepsilon \gamma^5 \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} Dv = v \varepsilon \gamma^5 Dv = \frac{i}{R} \left(E^a \pm \frac{R}{2} \Omega^{a3} \right) v \varepsilon \gamma_a v. \quad (\text{C.10})$$

C.2 Identities involving $\vartheta^{\alpha a'}$ and the simplified form of the $\text{OSp}(6|4)/\text{U}(3) \times \text{SO}(1,3)$ supergeometry

Using the definition of \mathcal{M}_{24} , eq. (B.2), and the fact that

$$[\gamma^{012}, \gamma^{a'}] = 0 \quad (\text{C.11})$$

we find that

$$(\vartheta \gamma' \gamma^5 \mathcal{M}_{24}^2)_{\beta b'} = 0 \quad (\mathcal{M}_{24}^2 \gamma' \vartheta)^{\alpha a'} = 0, \quad (\text{C.12})$$

where γ' is any product of the gamma-matrices that commutes with $\gamma = \gamma^{012}$, e.g. any product of $\gamma^{a'}$ and γ^a . A slightly longer computation, using the fact that

$$\gamma^3 \vartheta = \pm \gamma^3 \gamma^{012} \vartheta = \pm i \gamma^5 \vartheta, \quad \vartheta \gamma^3 = \mp i \vartheta \gamma^5 \quad \text{for } \vartheta = \frac{1}{2} (1 \pm \gamma) \vartheta, \quad (\text{C.13})$$

shows that with this projection of the ϑ s

$$\mathcal{M}_{24}^4 = 0. \quad (\text{C.14})$$

Using the identity

$$\vartheta^{\alpha a'} \vartheta^{\beta b'} \delta_{a' b'} = -\frac{1}{4} ((1 \pm \gamma) C^{-1})^{\alpha \beta} \vartheta \vartheta, \quad (\text{C.15})$$

where $\vartheta \vartheta \equiv \vartheta^{\alpha a'} C_{\alpha \beta} \vartheta^{\beta b'} \delta_{a' b'}$, one can further show that

$$(\mathcal{M}_{24}^2 D_{24} \vartheta)^{\alpha a'} = \frac{6i}{R^2} \left(e^b \pm \frac{R}{2} \omega^{b3} \right) (\gamma_b \vartheta)^{\alpha a'} \vartheta \vartheta, \quad (\text{C.16})$$

where the covariant derivative D_{24} , defined in (B.3), becomes

$$D_{24}\vartheta = \mathcal{P}_6 \left(d + \frac{i}{R} \left(e^a \pm \frac{R}{2} \omega^{a3} \right) \gamma^5 \gamma_a \mp \frac{1}{R} e^3 + \frac{i}{R} e^{a'} \gamma_{a'} - \frac{1}{4} \omega^{ab} \gamma_{ab} - \frac{1}{4} \omega^{a'b'} \gamma_{a'b'} \right) \vartheta. \quad (\text{C.17})$$

This gives

$$\vartheta \gamma^a \left(1 + \frac{1}{12} \mathcal{M}_{24}^2 \right) D_{24} \vartheta = \vartheta \gamma^a D_{24} \vartheta + \frac{i}{2R^2} \left(e^a \pm \frac{R}{2} \omega^{a3} \right) (\vartheta \vartheta)^2. \quad (\text{C.18})$$

Using the above expressions one finds that the form of the $\text{OSp}(6|4)/\text{U}(3) \times \text{SO}(1,3)$ geometrical objects (B.1) simplify to

$$\begin{aligned} E^a &= e^a(x) + i\vartheta \gamma^a D_{24} \vartheta - \frac{1}{2R^2} \left(e^a \pm \frac{R}{2} \omega^{a3} \right) (\vartheta \vartheta)^2, \\ E^3 &= e^3(x), \\ E^{a'} &= e^{a'}(y) - \frac{1}{R} \left(e^a \pm \frac{R}{2} \omega^{a3} \right) \vartheta \gamma^{a'} \gamma_a \vartheta \\ E^{\alpha a'} &= (D_{24} \vartheta)^{\alpha a'} + \frac{i}{R^2} \left(e^b \pm \frac{R}{2} \omega^{b3} \right) (\gamma_b \vartheta)^{\alpha a'} \vartheta, \\ \Omega^{ab} &= \omega^{ab}(x) + \frac{2i}{R^2} \left(e^c \pm \frac{R}{2} \omega^{c3} \right) \vartheta \gamma^{ab} \vartheta, \\ \Omega^{a3} &= \omega^{a3}(x) \mp \frac{2i}{R} \vartheta \gamma^a D_{24} \vartheta \pm \frac{1}{R^3} \left(e^a \pm \frac{R}{2} \omega^{a3} \right) (\vartheta \vartheta)^2, \\ \Omega^{a'b'} &= \omega^{a'b'}(y) - \frac{i}{R^2} \left(e^a \pm \frac{R}{2} \omega^{a3} \right) \vartheta (\gamma^{a'b'} - i J^{a'b'} \gamma^7) \gamma_a \vartheta, \\ A &= A(y) - \frac{1}{R^2} \left(e^a \pm \frac{R}{2} \omega^{a3} \right) \vartheta \gamma^7 \gamma_a \vartheta, \end{aligned} \quad (\text{C.19})$$

and in particular

$$E^a \pm \frac{R}{2} \Omega^{a3} = e^a(x) \pm \frac{R}{2} \omega^{a3}(x). \quad (\text{C.20})$$

Thus, in the chosen κ -symmetry gauge the $\text{OSp}(6|4)/\text{U}(3) \times \text{SO}(1,3)$ supercoset geometry depends on the fermionic coordinates only up to the 4th power.

Note that in all the above expressions the components $e^a(x)$ ($a = 0, 1, 2$) and $R/2 \omega^{a3}(x)$ of the AdS_4 vielbein and connection appear only in the combination $e^a(x) \pm R/2 \omega^{a3}(x)$. This combination has a very clear geometrical meaning. In the case, when the indices $a = 0, 1, 2$ label the directions of the 3d Minkowski slice of the AdS_4 , $e^a(x) \pm R/2 \omega^{a3}(x)$ corresponds to the generator $\Pi_a = P_a \mp 1/2 M_{a3}$ of the Poincaré translations ($[\Pi_a, \Pi_b] = 0$) along the 3d Minkowski boundary which is the linear combination of boosts and Lorentz rotations in AdS_4 (see [19] for more details). More precisely, $e^a(x) - R/2 \omega^{a3}(x)$ corresponds to the Poincaré translation, while $e^a(x) + R/2 \omega^{a3}(x)$ corresponds to the conformal boosts in M_3 , or vice versa, depending on the orientation.

When the AdS_4 metric is chosen in the form (3.1) the vielbein $e^a(x)$ and the connection $\omega^{a3}(x)$ are proportional to each other, namely,

$$e^a = -\frac{R}{2} \omega^{a3}. \quad (\text{C.21})$$

Actually, this relation can be imposed for any form of the metric by performing an appropriate $SO(1, 3)$ transformation of the AdS_4 vielbein and connection.

References

- [1] J. Bagger and N. Lambert, *Modeling multiple M2's*, *Phys. Rev. D* **75** (2007) 045020 [[hep-th/0611108](#)] [[SPIRES](#)]; *Gauge symmetry and supersymmetry of multiple M2-branes*, *Phys. Rev. D* **77** (2008) 065008 [[arXiv:0711.0955](#)] [[SPIRES](#)]; *Comments on multiple M2-branes*, *JHEP* **02** (2008) 105 [[arXiv:0712.3738](#)] [[SPIRES](#)].
- [2] A. Gustavsson, *Algebraic structures on parallel M2-branes*, *Nucl. Phys. B* **811** (2009) 66 [[arXiv:0709.1260](#)] [[SPIRES](#)].
- [3] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, *$N = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, *JHEP* **10** (2008) 091 [[arXiv:0806.1218](#)] [[SPIRES](#)].
- [4] J. Gomis, D. Sorokin and L. Wulff, *The complete $AdS_4 \times CP^3$ superspace for the type IIA superstring and D-branes*, *JHEP* **03** (2009) 015 [[arXiv:0811.1566](#)] [[SPIRES](#)].
- [5] F. Giani and M. Pernici, *$N = 2$ supergravity in ten-dimensions*, *Phys. Rev. D* **30** (1984) 325 [[SPIRES](#)].
- [6] B.E.W. Nilsson and C.N. Pope, *Hopf fibration of eleven-dimensional supergravity*, *Class. Quant. Grav.* **1** (1984) 499 [[SPIRES](#)].
- [7] D.P. Sorokin, V.I. Tkach and D.V. Volkov, *Kaluza-Klein theories and spontaneous compactification mechanisms of extra space dimensions*, in *Proceedings of the third seminar on quantum gravity*, Moscow USSR October 23–25 1984, pg. 376 [[SPIRES](#)]; *On the relationship between compactified vacua of $D = 11$ and $D = 10$ supergravities*, *Phys. Lett. B* **161** (1985) 301 [[SPIRES](#)].
- [8] S. Watamura, *Spontaneous compactification and $CP_N: SU(3) \times SU(2) \times U(1)$, $\sin^2\theta_W$, g_3/g_2 and $SU(3)$ triplet chiral fermions in four-dimensions*, *Phys. Lett. B* **136** (1984) 245 [[SPIRES](#)].
- [9] G. Arutyunov and S. Frolov, *Superstrings on $AdS_4 \times CP^3$ as a coset σ -model*, *JHEP* **09** (2008) 129 [[arXiv:0806.4940](#)] [[SPIRES](#)].
- [10] B.J. Stefanski, *Green-Schwarz action for type IIA strings on $AdS_4 \times CP^3$* , *Nucl. Phys. B* **808** (2009) 80 [[arXiv:0806.4948](#)] [[SPIRES](#)].
- [11] P. Fré and P.A. Grassi, *Pure spinor formalism for $OSp(N|4)$ backgrounds*, [arXiv:0807.0044](#) [[SPIRES](#)].
- [12] G. Bonelli, P.A. Grassi and H. Safaai, *Exploring pure spinor string theory on $AdS_4 \times CP^3$* , *JHEP* **10** (2008) 085 [[arXiv:0808.1051](#)] [[SPIRES](#)].
- [13] R. D'Auria, P. Fré, P.A. Grassi and M. Trigiante, *Superstrings on $AdS_4 \times CP^3$ from supergravity*, [arXiv:0808.1282](#) [[SPIRES](#)].
- [14] R. Kallosh, *Superconformal actions in Killing gauge*, [hep-th/9807206](#) [[SPIRES](#)].
- [15] I. Pesando, *A kappa gauge fixed type IIB superstring action on $AdS_5 \times S^5$* , *JHEP* **11** (1998) 002 [[hep-th/9808020](#)] [[SPIRES](#)].

- [16] R. Kallosh and J. Rahmfeld, *The GS string action on $AdS_5 \times S^5$* , *Phys. Lett. B* **443** (1998) 143 [[hep-th/9808038](#)] [[SPIRES](#)].
- [17] R. Kallosh and A.A. Tseytlin, *Simplifying superstring action on $AdS_5 \times S^5$* , *JHEP* **10** (1998) 016 [[hep-th/9808088](#)] [[SPIRES](#)].
- [18] P. Claus, *Super M-brane actions in $AdS_4 \times S^7$ and $AdS_7 \times S^4$* , *Phys. Rev. D* **59** (1999) 066003 [[hep-th/9809045](#)] [[SPIRES](#)].
- [19] P. Pasti, D.P. Sorokin and M. Tonin, *On gauge-fixed superbrane actions in AdS superbackgrounds*, *Phys. Lett. B* **447** (1999) 251 [[hep-th/9809213](#)] [[SPIRES](#)].
- [20] D.V. Uvarov, *$AdS_4 \times CP^3$ superstring and $D = 3$ $N = 6$ superconformal symmetry*, *Phys. Rev. D* **79** (2009) 106007 [[arXiv:0811.2813](#)] [[SPIRES](#)].
- [21] K. Zarembo, *Worldsheet spectrum in AdS_4/CFT_3 correspondence*, [arXiv:0903.1747](#) [[SPIRES](#)].
- [22] R. Ricci, A.A. Tseytlin and M. Wolf, *On T-duality and integrability for strings on AdS backgrounds*, *JHEP* **12** (2007) 082 [[arXiv:0711.0707](#)] [[SPIRES](#)].
- [23] N. Berkovits and J. Maldacena, *Fermionic T-duality, dual superconformal symmetry and the amplitude/Wilson loop connection*, *JHEP* **09** (2008) 062 [[arXiv:0807.3196](#)] [[SPIRES](#)].
- [24] N. Beisert, R. Ricci, A.A. Tseytlin and M. Wolf, *Dual superconformal symmetry from $AdS_5 \times S^5$ superstring integrability*, *Phys. Rev. D* **78** (2008) 126004 [[arXiv:0807.3228](#)] [[SPIRES](#)].
- [25] I. Adam, A. Dekel and Y. Oz, *On integrable backgrounds self-dual under fermionic T-duality*, *JHEP* **04** (2009) 120 [[arXiv:0902.3805](#)] [[SPIRES](#)].
- [26] B. de Wit, K. Peeters and J. Plefka, *Superspace geometry for supermembrane backgrounds*, *Nucl. Phys. B* **532** (1998) 99 [[hep-th/9803209](#)] [[SPIRES](#)];
B. de Wit, K. Peeters, J. Plefka and A. Sevrin, *The M-theory two-brane in $AdS_4 \times S^7$ and $AdS_7 \times S^4$* , *Phys. Lett. B* **443** (1998) 153 [[hep-th/9808052](#)] [[SPIRES](#)].
- [27] R.R. Metsaev and A.A. Tseytlin, *Supersymmetric D3 brane action in $AdS_5 \times S^5$* , *Phys. Lett. B* **436** (1998) 281 [[hep-th/9806095](#)] [[SPIRES](#)].
- [28] M.J. Duff, *Near-horizon brane-scan revived*, *Nucl. Phys. B* **810** (2009) 193 [[arXiv:0804.3675](#)] [[SPIRES](#)].
- [29] N. Drukker, J. Gomis and D. Young, *Vortex loop operators, M2-branes and holography*, *JHEP* **03** (2009) 004 [[arXiv:0810.4344](#)] [[SPIRES](#)].
- [30] D. Berenstein and D. Trancanelli, *Three-dimensional $N = 6$ SCFT's and their membrane dynamics*, *Phys. Rev. D* **78** (2008) 106009 [[arXiv:0808.2503](#)] [[SPIRES](#)].
- [31] N. Drukker, J. Plefka and D. Young, *Wilson loops in 3-dimensional $N = 6$ supersymmetric Chern-Simons theory and their string theory duals*, *JHEP* **11** (2008) 019 [[arXiv:0809.2787](#)] [[SPIRES](#)].
- [32] B. Chen and J.-B. Wu, *Supersymmetric Wilson loops in $N = 6$ super Chern-Simons-matter theory*, [arXiv:0809.2863](#) [[SPIRES](#)].
- [33] J. Kluson and K.L. Panigrahi, *Defects and Wilson loops in 3D QFT from D-branes in $AdS_4 \times CP^3$* , [arXiv:0809.3355](#) [[SPIRES](#)].

- [34] S.-J. Rey, T. Suyama and S. Yamaguchi, *Wilson loops in superconformal Chern-Simons theory and fundamental strings in Anti-de Sitter supergravity dual*, *JHEP* **03** (2009) 127 [[arXiv:0809.3786](#)] [[SPIRES](#)].
- [35] P.S. Howe and E. Sezgin, *The supermembrane revisited*, *Class. Quant. Grav.* **22** (2005) 2167 [[hep-th/0412245](#)] [[SPIRES](#)].
- [36] G. Dall'Agata et al., *The $OSp(8|4)$ singleton action from the supermembrane*, *Nucl. Phys. B* **542** (1999) 157 [[hep-th/9807115](#)] [[SPIRES](#)].
- [37] N. Berkovits, *Covariant quantization of the supermembrane*, *JHEP* **09** (2002) 051 [[hep-th/0201151](#)] [[SPIRES](#)].
- [38] P. Fré' and P.A. Grassi, *Pure spinors, free differential algebras and the supermembrane*, *Nucl. Phys. B* **763** (2007) 1 [[hep-th/0606171](#)] [[SPIRES](#)].
- [39] T.H. Buscher, *Path integral derivation of quantum duality in nonlinear σ -models*, *Phys. Lett. B* **201** (1988) 466 [[SPIRES](#)].
- [40] A.S. Schwarz and A.A. Tseytlin, *Dilaton shift under duality and torsion of elliptic complex*, *Nucl. Phys. B* **399** (1993) 691 [[hep-th/9210015](#)] [[SPIRES](#)].
- [41] J.A. Minahan and K. Zarembo, *The Bethe ansatz for superconformal Chern-Simons*, *JHEP* **09** (2008) 040 [[arXiv:0806.3951](#)] [[SPIRES](#)].
- [42] D. Gaiotto, S. Giombi and X. Yin, *Spin chains in $N = 6$ superconformal Chern-Simons-matter theory*, *JHEP* **04** (2009) 066 [[arXiv:0806.4589](#)] [[SPIRES](#)].
- [43] D. Bak and S.-J. Rey, *Integrable spin chain in superconformal Chern-Simons theory*, *JHEP* **10** (2008) 053 [[arXiv:0807.2063](#)] [[SPIRES](#)].
- [44] N. Gromov and P. Vieira, *The all loop AdS_4/CFT_3 Bethe ansatz*, *JHEP* **01** (2009) 016 [[arXiv:0807.0777](#)] [[SPIRES](#)].
- [45] N. Gromov, V. Kazakov and P. Vieira, *Integrability for the full spectrum of planar AdS/CFT* , [arXiv:0901.3753](#) [[SPIRES](#)].
- [46] C. Ahn and R.I. Nepomechie, *$N = 6$ super Chern-Simons theory S -matrix and all-loop Bethe ansatz equations*, *JHEP* **09** (2008) 010 [[arXiv:0807.1924](#)] [[SPIRES](#)]; *An alternative S -matrix for $N = 6$ Chern-Simons theory?*, *JHEP* **03** (2009) 068 [[arXiv:0810.1915](#)] [[SPIRES](#)]; *Two-loop test of the $N = 6$ Chern-Simons theory S -matrix*, *JHEP* **03** (2009) 144 [[arXiv:0901.3334](#)] [[SPIRES](#)].
- [47] C. Kristjansen, M. Orselli and K. Zoubos, *Non-planar ABJM theory and integrability*, *JHEP* **03** (2009) 037 [[arXiv:0811.2150](#)] [[SPIRES](#)].
- [48] B.I. Zwiebel, *Two-loop integrability of planar $N = 6$ superconformal Chern-Simons theory*, [arXiv:0901.0411](#) [[SPIRES](#)].
- [49] J.A. Minahan, W. Schulgin and K. Zarembo, *Two loop integrability for Chern-Simons theories with $N = 6$ supersymmetry*, *JHEP* **03** (2009) 057 [[arXiv:0901.1142](#)] [[SPIRES](#)].
- [50] T. Nishioka and T. Takayanagi, *On type IIA Penrose limit and $N = 6$ Chern-Simons theories*, *JHEP* **08** (2008) 001 [[arXiv:0806.3391](#)] [[SPIRES](#)].
- [51] G. Grignani, T. Harmark and M. Orselli, *The $SU(2) \times SU(2)$ sector in the string dual of $N = 6$ superconformal Chern-Simons theory*, *Nucl. Phys. B* **810** (2009) 115 [[arXiv:0806.4959](#)] [[SPIRES](#)].

- [52] G. Grignani, T. Harmark, M. Orselli and G.W. Semenoff, *Finite size giant magnons in the string dual of $N = 6$ superconformal Chern-Simons theory*, *JHEP* **12** (2008) 008 [[arXiv:0807.0205](#)] [[SPIRES](#)].
- [53] D. Astolfi, V.G.M. Puletti, G. Grignani, T. Harmark and M. Orselli, *Finite-size corrections in the $SU(2) \times SU(2)$ sector of type IIA string theory on $AdS_4 \times CP^3$* , *Nucl. Phys. B* **810** (2009) 150 [[arXiv:0807.1527](#)] [[SPIRES](#)].
- [54] B.-H. Lee, K.L. Panigrahi and C. Park, *Spiky strings on $AdS_4 \times CP^3$* , *JHEP* **11** (2008) 066 [[arXiv:0807.2559](#)] [[SPIRES](#)].
- [55] L.F. Alday and J.M. Maldacena, *Comments on operators with large spin*, *JHEP* **11** (2007) 019 [[arXiv:0708.0672](#)] [[SPIRES](#)].
- [56] T. McLoughlin and R. Roiban, *Spinning strings at one-loop in $AdS_4 \times P^3$* , *JHEP* **12** (2008) 101 [[arXiv:0807.3965](#)] [[SPIRES](#)].
- [57] L.F. Alday, G. Arutyunov and D. Bykov, *Semiclassical quantization of spinning strings in $AdS_4 \times CP^3$* , *JHEP* **11** (2008) 089 [[arXiv:0807.4400](#)] [[SPIRES](#)].
- [58] C. Krishnan, *AdS_4/CFT_3 at one loop*, *JHEP* **09** (2008) 092 [[arXiv:0807.4561](#)] [[SPIRES](#)].
- [59] T. McLoughlin, R. Roiban and A.A. Tseytlin, *Quantum spinning strings in $AdS_4 \times CP^3$: testing the Bethe ansatz proposal*, *JHEP* **11** (2008) 069 [[arXiv:0809.4038](#)] [[SPIRES](#)].
- [60] P. Sundin, *The $AdS_4 \times CP^3$ string and its Bethe equations in the near plane wave limit*, *JHEP* **02** (2009) 046 [[arXiv:0811.2775](#)] [[SPIRES](#)].
- [61] M.C. Abbott and I. Aniceto, *Giant magnons in $AdS_4 \times CP^3$: embeddings, charges and a Hamiltonian*, [arXiv:0811.2423](#) [[SPIRES](#)];
M.C. Abbott, I. Aniceto and O.O. Sax, *Dyonic giant magnons in CP^3 : strings and curves at finite J* , [arXiv:0903.3365](#) [[SPIRES](#)].
- [62] A. Hamilton, J. Murugan, A. Prinsloo and M. Strydom, *A note on dual giant gravitons in $AdS_4 \times CP^3$* , [arXiv:0901.0009](#) [[SPIRES](#)].
- [63] O. Bergman and S. Hirano, *Anomalous radius shift in AdS_4/CFT_3* , *JHEP* **07** (2009) 016 [[arXiv:0902.1743](#)] [[SPIRES](#)].
- [64] T.J. Hollowood and J.L. Miramontes, *Magnons, their solitonic avatars and the Pohlmeyer reduction*, *JHEP* **04** (2009) 060 [[arXiv:0902.2405](#)] [[SPIRES](#)].
- [65] C. Kalousios, M. Spradlin and A. Volovich, *Dyonic giant magnons on CP^3* , *JHEP* **07** (2009) 006 [[arXiv:0902.3179](#)] [[SPIRES](#)].
- [66] R. Suzuki, *Giant magnons on CP^3 by dressing method*, *JHEP* **05** (2009) 079 [[arXiv:0902.3368](#)] [[SPIRES](#)].
- [67] M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell and A. Westerberg, *The Dirichlet super- p -branes in ten-dimensional type IIA and IIB supergravity*, *Nucl. Phys. B* **490** (1997) 179 [[hep-th/9611159](#)] [[SPIRES](#)].
- [68] M. Aganagic, C. Popescu and J.H. Schwarz, *D-brane actions with local kappa symmetry*, *Phys. Lett. B* **393** (1997) 311 [[hep-th/9610249](#)] [[SPIRES](#)].
- [69] E. Bergshoeff and P.K. Townsend, *Super D-branes*, *Nucl. Phys. B* **490** (1997) 145 [[hep-th/9611173](#)] [[SPIRES](#)].
- [70] I.L. Buchbinder et al., *ABJM models in $N = 3$ harmonic superspace*, *JHEP* **03** (2009) 096 [[arXiv:0811.4774](#)] [[SPIRES](#)].